Entropy Generation in Electro-Osmotic Flow in Microchannels

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ABSTRACT
An analysis has been provided for the entropy generation in thermally fully developed electro-osmotically generated flow in a parallel plate microchannel in terms of Brinkman number, Peclet number, dimensionless joule heating parameter, dimensionless viscous heating parameter as well as physical properties of the fluid under imposed constant wall heat flux boundary condition. Such a flow is established not by an imposed pressure gradient, but by a voltage potential gradient along the length of the tube. The solution for momentum and energy equations is used to get the exact solution for the dimensionless entropy generation number. This analysis assumes no pressure-driven component to the velocity field and constant fluid properties. Different variables namely, dimensionless joule heating parameter, viscous heating parameter, Peclet number and Brinkman number have been identified from the dimensionless entropy generation number equation. Various results for dimensionless entropy generation number, Bejan number, Irreversibility ratio, entropy generation due to fluid friction etc. are presented.

NOMENCLATURE

α	Channel gap half-width
Be	Bejan Number
Br	Brinkman number
C	Fluid specific heat
Dh	Channel hydraulic diameter
k	Thermal conductivity
N_S	Dimensionless entropy generation number
N_F	Entropy gen eration due to the fluid friction
N_C	Entropy generation due to heat transfer in the axial direction
N_Y	Entropy generation due to heat transfer in the transverse direction
Pe	Peclet number
q''_w	Wall heat flux
s_e	Volumetric Joule heating
S	Dimensionless joule heating parameter
S_v	Dimensionless viscous heating parameter
T	Absolute temperature
T_m	Mixed mean temperature
T_w	Channel wall temperature
u	Local fluid velocity
u_{max}	Maximum possible electro-osmotic velocity
$\bar{u}$	Average velocity
U	Normalized local velocity, $u/\bar{u}$
w	Parallel plate channel width
x	Streamwise coordinate

y	Wall-normal coordinate
Y	Normalized wall-normal coordinate, y/a

Greek symbols

α	Thermal diffusivity
ε	Fluid dielectric constant
μ	Fluid dynamic viscosity
φ	Applied potential field
Φ	Irreversibility ratio
ρ	Fluid density
θ	Normalized temperature
θ_w	Normalized wall temperature
λ	Debye length
ψ	Wall zeta potential

INTRODUCTION

Electro-osmosis is the bulk movement of liquid relative to a stationary surface due to an externally applied electric field. Reuss [1] first observed and reported this. A solid substance will acquire a relative electric charge when in contact with an aqueous electrolytic solution. This will influence the charge distribution in the solution. Ions of opposite charge (counterions) to that of the surface are attracted towards the surface and ions of the same charge (coions) are repelled from the surface. Thus, in a region close to the charged surface called the electric double layer [EDL] there will be an excess of counter ions over coions. The charge distribution in the fluid therefore falls from its maximum near the wall to a zero charge in the fluid core. The thickness of the EDL is characterized by the Debye length, which is the wall-normal distance over which the net charge has decreased from the charge magnitude near the channel surface to $1/e$ (37%) of the surface charge. The positively charged cations and solvent molecules strongly adsorbed at the wall remain stationary under the influence of an electric potential in the streamwise direction, while the mobile cations in the EDL very near the tube walls will migrate toward the cathode due to the excess charge in the layer.

Yang and Li [2] investigated electrokinetic effects induced in a pressure-driven flow on the frictional and heat transfer characteristics for both round and rectangular microchannels. Knox [3] studied the influence of Joule heating in capillary electrophoresis. The thermally fully-developed flow electro-osmotic flow in circular microtubes and parallel plate microchannels was considered by Maynes and Webb [4,5] for the case of negligible viscous dissipation.

Exergy analysis is pure thermodynamics. It relies on the laws of thermodynamics to establish the theoretical limit of ideal or reversible operation and the extent to which the operation of the given system departs from the ideal. The departure is measured by the calculated quantity called destroyed exergy or irreversibility. This quantity is proportional to the generated entropy. Exergy is the thermodynamic property that describes the useful energy content or the work producing potential of substances and streams. In real systems exergy is always destroyed partially or totally when components and streams interact. The minimization of entropy generation requires the use of more than thermodynamics: fluid mechanics, heat and mass transfer, materials, constraints, and geometry are also needed in order to establish the relationships between the physical configuration and the destruction of exergy. Reduction in exergy destruction is pursued through changes in configuration. In the field of heat transfer, the entropy generation minimization method brings out the inherent competition between heat transfer and fluid flow irreversibilities in the optimization of devices subjected to overall constraints.

A study of thermodynamic irreversibilities is an important factor in the design of thermal systems. Entropy is a measure of molecular disorder within a system, which translates into the amount of energy not available to be converted into work. Therefore, the main objective is to determine possible ways of minimizing it. The method utilized to achieve this purpose is known as Entropy Generation Minimization or thermodynamic optimization. This method is a combination of the basic principles of thermodynamics, heat transfer and fluid mechanics in order to analyze the irreversibilities present in real devices concentrating special attention in finding the physical quantities that can be modified to optimize their specific performance.

No studies have appeared in the literature that specifically address the entropy generation for purely electro-osmotically driven flow in a parallel plate microchannel. The present work has been undertaken in order to study the entropy generation for fully-developed electro-osmotic flow in a parallel plate microchannel for constant wall heat flux boundary condition. This analysis assumes no pressure-driven component to the velocity field and constant fluid properties.

MATHEMATICAL FORMULATION AND ANALYSIS

Consider a fully developed electro-osmotically driven flow of an incompressible fluid in a parallel plate microchannel of infinite width with coordinates as defined in Figure 1.

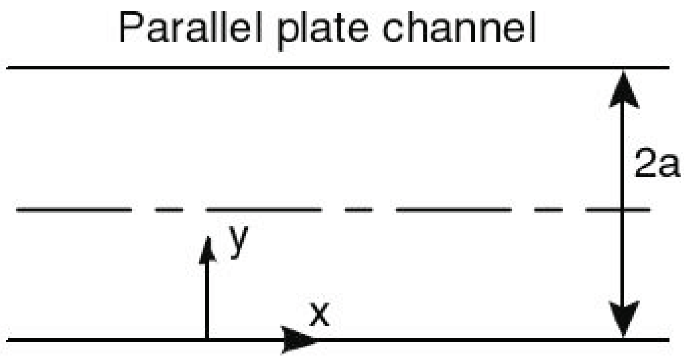

Figure 1. Coordinate system for Parallel plate microchannel

Momentum transport

For steady flow without an applied pressure gradient the streamwise momentum equation reduces [4] to

$$\mu\left(\frac{d^2u}{dy^2}\right)+\varepsilon\left(\frac{d^2\psi}{dy^2}\right)\frac{d\varphi}{dx}=0 \tag{1}$$

Here, μ is the fluid viscosity, ε is the dielectric constant, ϕ is the applied potential field and $\psi(y)$ is the access charge distribution.

For low wall potential, the Debye–Huckel linearization is valid [6], and the excess charge distribution may be expressed explicitly as a function only of the zeta potential, the Debye length λ and the wall-normal coordinate y. For such a condition, the momentum equation (1) may be solved subject to boundary conditions with no slip at the wall and zero shear stress at the centerline, the fully-developed electro-osmotic velocity distribution for the parallel plate can be expressed as [6]

$$\frac{u}{u_{\max}}=1-Y\cdot Z\cdot e^{-Z}-e^{-Y\cdot Z} \quad (Y\le 1) \tag{2}$$

where, $Y=y/a$ is the normalized wall-normal coordinate and $Z=a/\lambda$ is the relative channel height. The Debye length λ is a function of the electro-chemical characteristics of the liquid / channel interface, and rather difficult to measure. It may be estimated from the relation $\lambda=\left(\varepsilon RT/2F^2z^2c\right)^{1/2}$ where ε , T, R and F are the fluid permittivity, absolute temperature, universal gas constant, and Faraday's constant respectively. The parameters z and c are the valence number and the average molar concentration of ions in the liquid solution, respectively.

Energy equation

Given steady hydrodynamically fully developed flow with constant thermophysical properties, the energy equation simplifies [4] to

$$\frac{\partial^2 T}{\partial x^2}+\left(\frac{\partial^2 T}{\partial y^2}\right)=\frac{u}{\alpha}\frac{\partial T}{\partial x}-\frac{s}{k} \tag{3}$$

Here, T is the local temperature, α is the thermal diffusivity, k is the thermal conductivity and s is the volumetric Joule heating.

For constant surface heat flux boundary condition (q''_w = constant), we have:

$$\frac{\partial T}{\partial x}=\frac{dT_m}{dx}=\text{constant} \tag{4}$$

Furthermore, an energy balance on the fluid yields

$$\frac{\partial T}{\partial x}=\frac{4\cdot q''_w}{\rho\cdot\bar{u}\cdot C\cdot D_h}+\frac{s}{\rho\cdot\bar{u}\cdot C} \tag{5}$$

In the above equation, D_h is the channel hydraulic diameter, q''_w is the wall heat flux and C is the fluid specific heat.

The general expression for the temperature distribution for parallel plate microchannel was given by Maynes ans Webb [7] as:

$$\theta=\left\{1+S\cdot\left[1+S_v\cdot F(Z)\right]\right\}\cdot A_2(Y,Z)+S\cdot\left[A_1(Y)-S_v\cdot A_3\cdot(Y,Z)\right] \tag{7}$$

where, S = dimensionless joule heating parameter, S_v = dimensionless viscous heating parameter and $\theta=\frac{k\cdot T}{a\cdot q''_w}$ = normalized temperature. Also,

$$F(Z)=\frac{3\cdot e^{-2\cdot Z}}{2\cdot Z}-\frac{2\cdot e^{-2\cdot Z}}{Z}+e^{-2\cdot Z}+\frac{1}{2\cdot Z} \tag{8}$$

$$A_1(Y)=-\left(\frac{Y^2}{2}-Y\right) \tag{9}$$

$$A_2(Y,Z)=\frac{1}{Z^2}\left(1-e^{-Y\cdot Z}\right)-Y\left(1+\frac{e^{-Z}}{Z}-\frac{Z\cdot e^{-Z}}{2}\right)+\frac{Y^2}{2}-\frac{Y^3}{6}Z\cdot e^{-Z} \tag{10}$$

and

$$A_3(Y,Z)=\frac{1}{Z^2}\left[\frac{1}{4}\left(e^{-2\cdot Y\cdot Z}-1\right)+2\left(e^{-Z}-e^{-Z(Y+1)}\right)\right]-Y\left(e^{-2\cdot Z}+\frac{3\cdot e^{-2\cdot Z}}{2\cdot Z}\right) \tag{11}$$

Substituting the values of equations (8), (9), (10) and (11) into equation (7), we have

$$\theta=\left\{1+S\cdot\left[1+S_v\cdot\left(\frac{3\cdot e^{-2\cdot Z}}{2\cdot Z}-\frac{2\cdot e^{-2\cdot Z}}{Z}+e^{-2\cdot Z}+\frac{1}{2\cdot Z}\right)\right]\right\}\cdot$$

$$\left(\frac{1}{Z^2}\left(1-e^{-Y\cdot Z}\right)-Y\left(1+\frac{e^{-Z}}{Z}-\frac{Z\cdot e^{-Z}}{2}\right)+\frac{Y^2}{2}-\frac{Y^3}{6}Z\cdot e^{-Z}\right)+$$
$$S\cdot\left[\left(-\left(\frac{Y^2}{2}-Y\right)\right)-S_v\cdot\left(\frac{1}{Z^2}\left[\frac{1}{4}\left(e^{-2\cdot Y\cdot Z}-1\right)+2\left(e^{-Z}-e^{-Z(Y+1)}\right)\right]-Y\left(e^{-2\cdot Z}+\frac{3\cdot e^{-2\cdot Z}}{2\cdot Z}\right)\right)\right] \tag{12}$$

A study of thermodynamic irreversibilities is an important factor in the design of thermal systems. Entropy is a measure of molecular disorder within a system, which translates into the amount of energy not available to be converted into work. Therefore, the main objective is to determine possible ways of minimizing it. The method utilized to achieve this purpose is known as Entropy Generation Minimization or thermodynamic optimization. This method is a combination of the basic principles of thermodynamics, heat transfer and fluid mechanics in order to analyze the irreversibilities present in real devices concentrating special attention in finding the physical quantities that can be modified to optimize their specific performance. Applications of second law analysis for channel and pipe may be found in references [8-11]. Entropy generation in a vertical concentric channel with variable viscosity was studied by Tasnim and Mahmud [12]. Haddad et. al. [13] investigated the entropy generation in a laminar forced convection in the entrance region of a concentric annulus. Nag and Kumar [14] performed second law analysis for convective heat transfer in rectangular ducts.

The equation for the entropy generation per unit volume is given by

$$S_G'''=\frac{\mu}{T_0}\left(\frac{\partial u}{\partial y}\right)^2+\frac{k}{T_0^2}\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{13}$$

In the above equation,

$$\frac{\mu}{T_0}\left(\frac{\partial u}{\partial y}\right)^2 = \text{friction Component and}$$

$$\frac{k}{T_0^2}\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right] = \text{heat flux component} \tag{14}$$

Here, $\frac{\partial T}{\partial x}$ = axial heat flux component and $\frac{\partial T}{\partial y}$ = transverse heat flux component

Multiplying both sides of equation (13) by $k\cdot T_0^2/q_w''^2$ to get the value of N_S ,

$$\left(\frac{k\cdot T_0^2}{q_w''^2}\right)S_G'''=N_S=\frac{\mu\cdot k\cdot T_0}{q_w''^2}\left(\frac{\partial u}{\partial y}\right)^2+\frac{k^2}{q_w''^2}\left(\frac{\partial T}{\partial x}\right)^2+\frac{k^2}{q_w''^2}\left(\frac{\partial T}{\partial y}\right)^2 \tag{15}$$

$$N_S=N_F+N_C+N_Y \tag{16}$$

In the above equations,

N_S = dimensionless entropy generation number

$N_F = \frac{\mu \cdot k \cdot T_0}{q_w''^2}\left(\frac{\partial u}{\partial y}\right)^2$ = entropy generation due to the fluid friction

$N_C = \frac{k^2}{q_w''^2}\left(\frac{\partial T}{\partial x}\right)^2$ = entropy generation due to heat transfer in the axial direction

$N_Y = \frac{k^2}{q_w''^2}\left(\frac{\partial T}{\partial y}\right)^2$ = entropy generation due to heat transfer in the transverse direction (17)

Brinkman number (Br) and Peclet number (Pe) are defined as,

$$Br = \frac{u^2_{\max} \cdot \mu \cdot k \cdot T_0}{q_w''^2 \cdot a^2} \quad \text{and}$$

$$Pe = \frac{\rho \cdot \bar{u} \cdot C \cdot D_h \cdot q_w''}{k} \tag{18}$$

Substituting values of Brinkman number (Br) and Peclet number (Pe) from equation (18) into equation (17),

$$\begin{aligned} N_S = {} & Br \cdot \left[Z^2 \cdot e^{-2 \cdot Y \cdot Z} - 2 \cdot Z^2 \cdot e^{-Z - Y \cdot Z} + Z^2 \cdot e^{-2 \cdot Z} \right] + \\ & \frac{1}{Pe^2} \cdot \left[16 \cdot q_w''^2 + s^2 \cdot D_h{}^2 + 8 \cdot q_w'' \cdot s \cdot D_h \right] + \left[Term1 \right]^2 \end{aligned} \tag{19}$$

where

$$\begin{aligned} Term1 = {} & \left(\frac{e^{-Y \cdot Z}}{Z} - 1 - \frac{e^{-Z}}{Z} + \frac{Z \cdot e^{-Z}}{2} + Y - \frac{Z \cdot Y^2 \cdot e^{-Z}}{2} \right) + \\ & \left(\frac{S \cdot e^{-Y \cdot Z}}{Z} - S - \frac{S \cdot e^{-Z}}{Z} + \frac{S \cdot Z \cdot e^{-Z}}{2} + S \cdot Y - \frac{S \cdot Y^2 \cdot Z \cdot e^{-Z}}{2} \right) + \\ & \left(\begin{aligned} & \frac{3 \cdot S \cdot S_V \cdot e^{-2 \cdot Z - Y \cdot Z}}{2 \cdot Z^2} - \frac{3 \cdot S \cdot S_V \cdot e^{-2 \cdot Z}}{2 \cdot Z} - \frac{3 \cdot S \cdot S_V \cdot e^{-3 \cdot Z}}{2 \cdot Z^2} - \\ & \frac{3 \cdot S \cdot S_V \cdot e^{-3 \cdot Z}}{4} + \frac{3 \cdot S \cdot S_V \cdot Y \cdot e^{-2 \cdot Z}}{2 \cdot Z} - \frac{3 \cdot S \cdot S_V \cdot Y^2 \cdot e^{-3 \cdot Z}}{4} \end{aligned} \right) - \\ & \left(\begin{aligned} & \frac{-2 \cdot S \cdot S_V \cdot e^{-Z - Y \cdot Z}}{Z^2} + \frac{2 \cdot S \cdot S_V \cdot e^{-Z}}{Z} + \frac{2 \cdot S \cdot S_V \cdot e^{-2 \cdot Z}}{Z^2} - \\ & \frac{S \cdot S_V \cdot e^{-2 \cdot Z}}{1} - \frac{2 \cdot S \cdot S_V \cdot Y \cdot e^{-Z}}{Z} + \frac{S \cdot S_V \cdot Y^2 \cdot e^{-2 \cdot Z}}{1} \end{aligned} \right) + \\ & \left(\frac{S \cdot S_V \cdot e^{-Z - Y \cdot Z}}{Z} - \frac{S \cdot S_V \cdot e^{-Z}}{1} - \frac{S \cdot S_V \cdot e^{-2 \cdot Z}}{Z} + \frac{S \cdot S_V \cdot Z \cdot e^{-2 \cdot Z}}{2} + \frac{S \cdot S_V \cdot Y \cdot e^{-Z}}{1} - \frac{S \cdot S_V \cdot Y^2 \cdot e^{-2 \cdot Z}}{2} \right) + \\ & \left(\frac{S \cdot S_V \cdot e^{-Y \cdot Z}}{2 \cdot Z^2} - \frac{S \cdot S_V}{2 \cdot Z} - \frac{S \cdot S_V \cdot e^{-Z}}{2 \cdot Z^2} + \frac{S \cdot S_V \cdot Z \cdot e^{-Z}}{4 \cdot Z} + \frac{S \cdot S_V \cdot Y}{2 \cdot Z} - \frac{S \cdot S_V \cdot Y^2 \cdot e^{-Z}}{4} \right) + \\ & \left(S - S \cdot Y - \frac{S \cdot S_V \cdot e^{-2 \cdot Y \cdot Z}}{2 \cdot Z} - \frac{2 \cdot S \cdot S_V \cdot e^{-Z - Y \cdot Z}}{Z} + \frac{S \cdot S_V \cdot e^{-2 \cdot Z}}{1} - \frac{3 \cdot S \cdot S_V \cdot e^{-2 \cdot Z}}{2 \cdot Z} \right) \end{aligned}$$

We also have:

$$N_F = Br \cdot \left[Z^2 \cdot e^{-2 \cdot Y \cdot Z} - 2 \cdot Z^2 \cdot e^{-Z-Y \cdot Z} + Z^2 \cdot e^{-2 \cdot Z} \right] \tag{20}$$

$$N_C = \frac{1}{Pe^2} \cdot \left[16 \cdot q_w''^2 + s^2 \cdot D_h{}^2 + 8 \cdot q_w'' \cdot s \cdot D_h \right] \tag{21}$$

and

$$N_Y = \left[\begin{array}{l} \left(\frac{e^{-Y \cdot Z}}{Z} - 1 - \frac{e^{-Z}}{Z} + \frac{Z \cdot e^{-Z}}{2} + Y - \frac{Z \cdot Y^2 \cdot e^{-Z}}{2} \right) + \\ \left(\frac{S \cdot e^{-Y \cdot Z}}{Z} - S - \frac{S \cdot e^{-Z}}{Z} + \frac{S \cdot Z \cdot e^{-Z}}{2} + S \cdot Y - \frac{S \cdot Y^2 \cdot Z \cdot e^{-Z}}{2} \right) + \\ \left(\begin{array}{l} \frac{3 \cdot S \cdot S_V \cdot e^{-2 \cdot Z - Y \cdot Z}}{2 \cdot Z^2} - \frac{3 \cdot S \cdot S_V \cdot e^{-2 \cdot Z}}{2 \cdot Z} - \frac{3 \cdot S \cdot S_V \cdot e^{-3 \cdot Z}}{2 \cdot Z^2} - \\ \frac{3 \cdot S \cdot S_V \cdot e^{-3 \cdot Z}}{4} + \frac{3 \cdot S \cdot S_V \cdot Y \cdot e^{-2 \cdot Z}}{2 \cdot Z} - \frac{3 \cdot S \cdot S_V \cdot Y^2 \cdot e^{-3 \cdot Z}}{4} \end{array} \right) - \\ \left(\begin{array}{l} \frac{-2 \cdot S \cdot S_V \cdot e^{-Z-Y \cdot Z}}{Z^2} + \frac{2 \cdot S \cdot S_V \cdot e^{-Z}}{Z} + \frac{2 \cdot S \cdot S_V \cdot e^{-2 \cdot Z}}{Z^2} - \\ \frac{S \cdot S_V \cdot e^{-2 \cdot Z}}{1} - \frac{2 \cdot S \cdot S_V \cdot Y \cdot e^{-Z}}{Z} + \frac{S \cdot S_V \cdot Y^2 \cdot e^{-2 \cdot Z}}{1} \end{array} \right) + \\ \left(\begin{array}{l} \frac{S \cdot S_V \cdot e^{-Z-Y \cdot Z}}{Z} - \frac{S \cdot S_V \cdot e^{-Z}}{1} - \frac{S \cdot S_V \cdot e^{-2 \cdot Z}}{Z} + \\ \frac{S \cdot S_V \cdot Z \cdot e^{-2 \cdot Z}}{2} + \frac{S \cdot S_V \cdot Y \cdot e^{-Z}}{1} - \frac{S \cdot S_V \cdot Y^2 \cdot e^{-2 \cdot Z}}{2} \end{array} \right) + \\ \left(\frac{S \cdot S_V \cdot e^{-Y \cdot Z}}{2 \cdot Z^2} - \frac{S \cdot S_V}{2 \cdot Z} - \frac{S \cdot S_V \cdot e^{-Z}}{2 \cdot Z^2} + \frac{S \cdot S_V \cdot Z \cdot e^{-Z}}{4 \cdot Z} + \frac{S \cdot S_V \cdot Y}{2 \cdot Z} - \frac{S \cdot S_V \cdot Y^2 \cdot e^{-Z}}{4} \right) + \\ \left(S - S \cdot Y - \frac{S \cdot S_V \cdot e^{-2 \cdot Y \cdot Z}}{2 \cdot Z} - \frac{2 \cdot S \cdot S_V \cdot e^{-Z-Y \cdot Z}}{Z} + \frac{S \cdot S_V \cdot e^{-2 \cdot Z}}{1} - \frac{3 \cdot S \cdot S_V \cdot e^{-2 \cdot Z}}{2 \cdot Z} \right) \end{array} \right]^2 \tag{22}$$

Irreversibility Ratio (Φ)

In our case, both the fluid friction and the heat transfer contribute to the rate of entropy generation. In order to assess which one among the fluid friction and heat transfer dominates, a criterion known as irreversibility ratio is defined by the following equation.

Irreversibility ratio (Φ) is the ratio of entropy generation due to the fluid friction to the total entropy generation due to heat transfer.

$$\Phi = \frac{\text{Fluid Friction Component}}{\text{Axial Heat Flux Component} + \text{Transverse Heat Flux Component}}$$

$$\Phi = \frac{N_F}{N_C + N_Y} \tag{23}$$

For $0 \le \Phi < 1$, the heat transfer dominates the irreversibility ratio and the fluid friction dominates when $\Phi > 1$. The case where both the heat transfer and the fluid friction have the same contribution for the entropy generation is characterized by $\Phi = 1$.

Bejan number (Be)

Bejan number is the ratio of heat transfer irreversibility to the total irreversibility due to fluid friction and heat transfer.

$$Be = \frac{Heat\ Flux\ \mathrm{Components}}{Fluid\ Friction\ Component\ +\ Heat\ Flux\ \mathrm{Components}}$$

$$Be = \frac{N_C + N_Y}{N_F + N_C + N_Y}$$

$$Be = \frac{1}{1 + \Phi} \tag{24}$$

Bejan number ranges from 0 to 1. Be = 0 is the limit where the irreversibility is dominated by fluid frictional effects and Be = 1 corresponds to the limit where the irreversibility due to heat transfer by virtue of finite temperature differences dominates.

Similarly, we define the following dimensionless ratios

$$G_{Friction} = \frac{Fluid\ Friction\ Component}{Fluid\ Friction\ Component\ +\ Heat\ Flux\ Components}$$

$$G_F = \frac{N_F}{N_F + N_C + N_Y} \tag{25}$$

and

$$G_{Heat\ \mathrm{Flux}} = \frac{Heat\ Flux\ \mathrm{Components}}{Fluid\ Friction\ Component\ +\ Heat\ Flux\ \mathrm{Components}}$$

$$G_H = \frac{N_C + N_Y}{N_F + N_C + N_Y} \tag{26}$$

DISCUSSION OF RESULTS

We have presented an analysis for the entropy generation on thermally fully developed, electro-osmotically generated flow for a parallel plate microchannel under constant wall heat flux boundary condition. The fluid is assumed to be water. The velocity and the temperature distributions are obtained analytically from the momentum equation and energy equation and used to compute the dimensionless entropy generation number (a), Irreversibility ratio (Φ), Bejan number (Be) and the dimensionless ratios G_F and G_H. These parameters are presented graphically for various values of dimensionless joule heating parameter (S), dimensionless viscous heating parameter (S_v), relative duct radius (Z), Brinkman number (Br) and Peclet number (Pe).

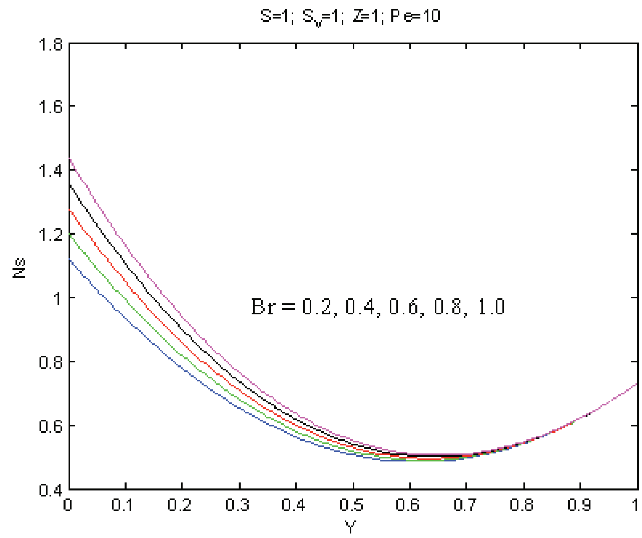

Figure 2. Ns Vs Y for S = 1; Sv = 1; Z = 1; Pe = 10 and Br = 0.2, 0.4, 0.6, 0.8 & 1.0

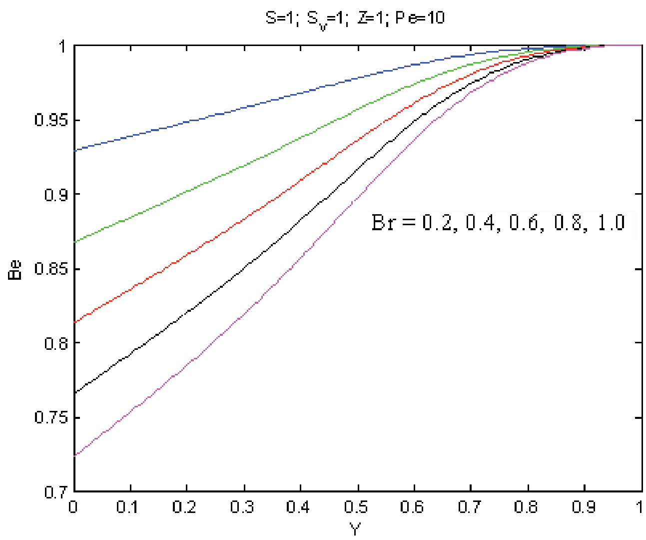

Figure 3. Be Vs Y for S = 1; Sv = 1; Z = 1; Pe = 10 and Br = 0.2, 0.4, 0.6, 0.8 & 1.0

Figure 2 shows the distribution of dimensionless entropy generation number Ns for constant values of S, Sv, Z and Pe. Here, Brinkman number Br is chosen as a parameter ranging from 0.2 to 1.0. From the figure, it is clear that Ns decreases as Y increases. Also as the value of Br increases, Ns also increases for a given value of Y. Figure 3 displays the Bejan number Be versus Y. Be is the ratio of heat transfer irreversibility to the total irreversibility due to fluid friction and heat transfer. Bejan number increases as the value of Y increases and approaches 1. Also as the value of Br increases, Be decreases. In Figure 4, Irreversibility ratio Φ is plotted versus Y. Irreversibility ratio Φ is the ratio of entropy generation due

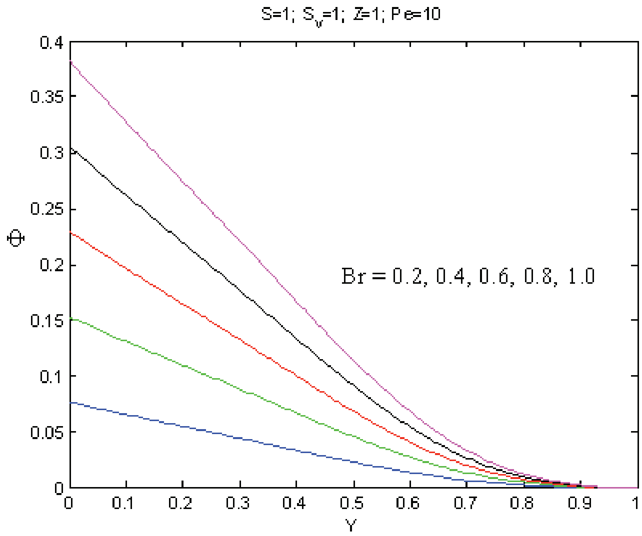

Figure 4. Φ Vs Y for S = 1; Sv = 1; Z = 1; Pe = 10 and Br = 0.2, 0.4, 0.6, 0.8 & 1.0

to the fluid friction to the total entropy generation due to heat transfer. Irreversibility ratio decreases as the value of Y increases and approaches 0. For $0 \leq \Phi < 1$, the heat transfer dominates the irreversibility ratio and the fluid friction dominates when $\Phi > 1$. Also it can be noted that as the value of Br increases, Φ increases. Figures 5 and 6 show the dimensionless ratios G_f and G_h versus Y. Looking at the equations and plots, the plots for G_f are similar to Φ and the plots for G_h are similar to Be. Figure 7 shows the entropy generation due to fluid friction N_f versus Y. It can be seen that as the value of Br increases, N_f increases for a particular value of Y. In Figure 8, N_h is plotted versus N_f. It can be seen that for a particular value of N_f, the value of N_h increases as Br increases.

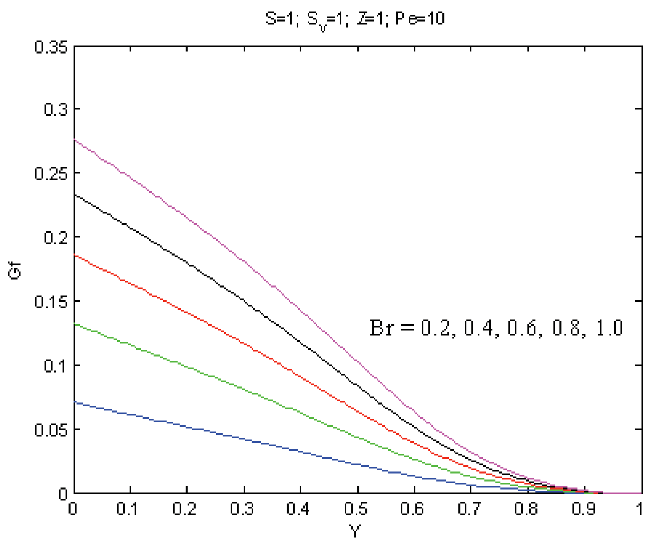

Figure 5. Gf Vs Y for S = 1; Sv = 1; Z = 1; Pe = 10 and Br = 0.2, 0.4, 0.6, 0.8 & 1.0

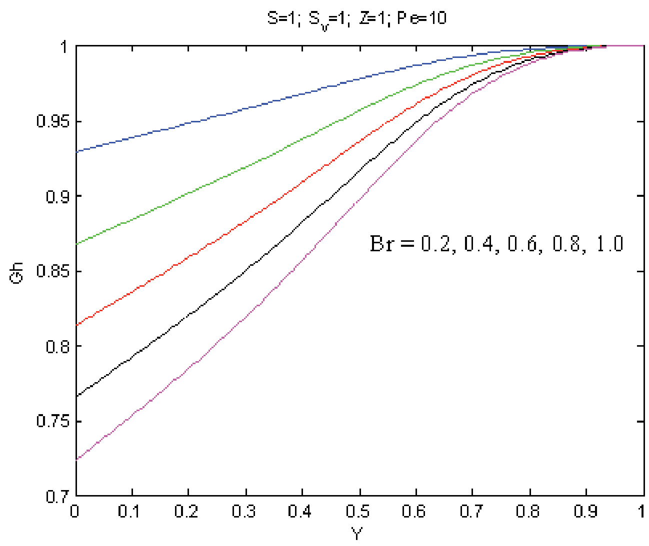

Figure 6. Gh Vs Y for S = 1; Sv = 1; Z = 1; Pe = 10 and Br = 0.2, 0.4, 0.6, 0.8 & 1.0

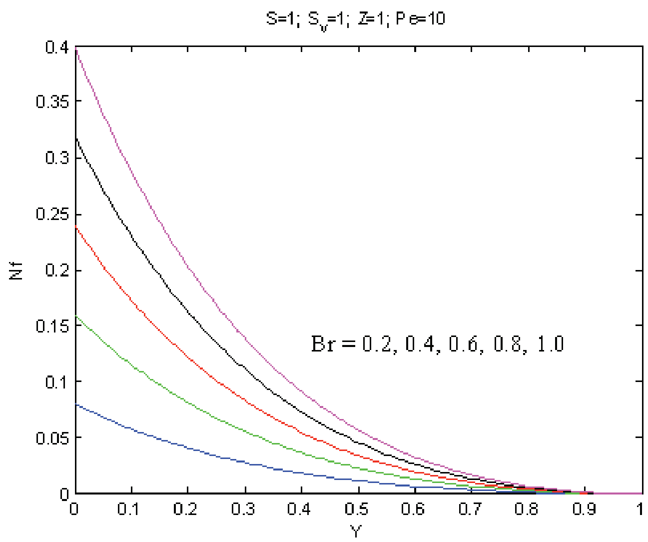

Figure 7. Nf Vs Y for S = 1; Sv = 1; Z = 1; Pe = 10 and Br = 0.2, 0.4, 0.6, 0.8 & 1.0

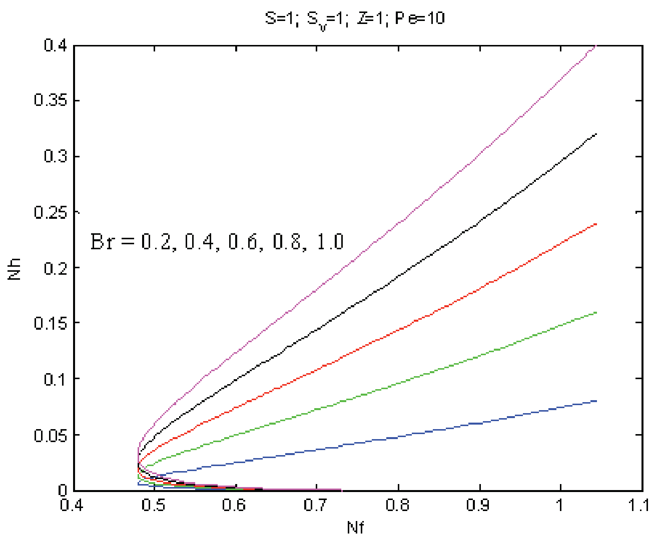

Figure 8. Nh Vs Nf for S = 1; Sv = 1; Z = 1; Pe = 10 and Br = 0.2, 0.4, 0.6, 0.8 & 1.0

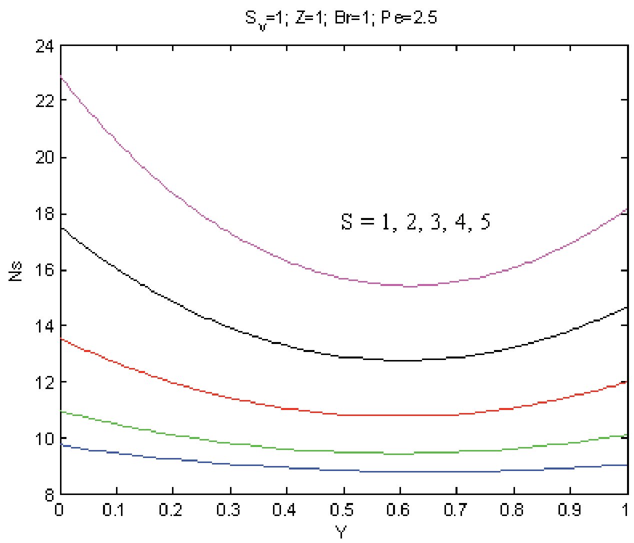

Figure 9. Ns Vs Y for Sv = 1; Z = 1; Br = 1; Pe = 2.5 and S = 1, 2, 3, 4 & 5

In Figure 9, Ns is plotted versus Y for constant values of Pe, Sv, Br and Z. Here dimensionless joule heating parameter S is chosen as a parameter ranging from 1 to 5. It can be seen that as the value of S increases, Ns also increases. From Figures 10 and 11, it is clear that as the value of S increases, Be increases and Φ decreases for any value of Y. In Figure 12, N_h is plotted versus Y. It can be seen here that as S increases, value of N_h also increases. In Figure 13, Ns is plotted versus Y for constant values of Pe, S, Br and Z. Here dimensionless viscous heating parameter Sv is chosen as a parameter ranging from 1 to 5. It can be seen that as the value of Sv increases, Ns decreases. From Figures 14 and 15, it is clear that as the value of Sv increases, Be decreases and Φ increases for any value of Y.

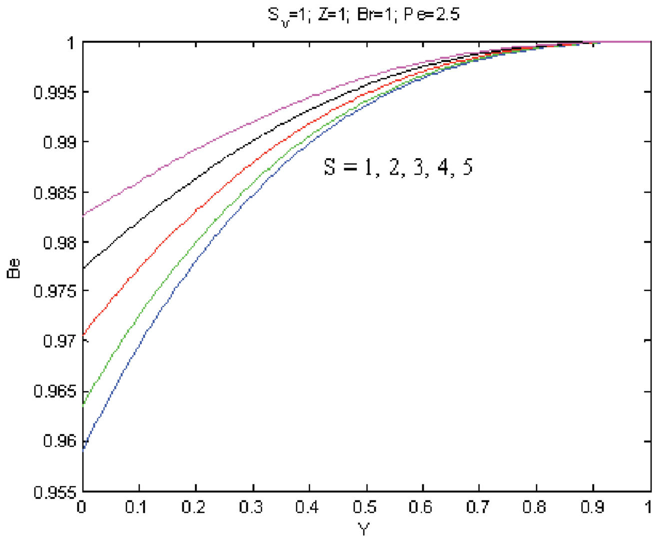

Figure 10. Be Vs Y for Sv = 1; Z = 1; Br = 1; Pe = 2.5 and S = 1, 2, 3, 4 & 5

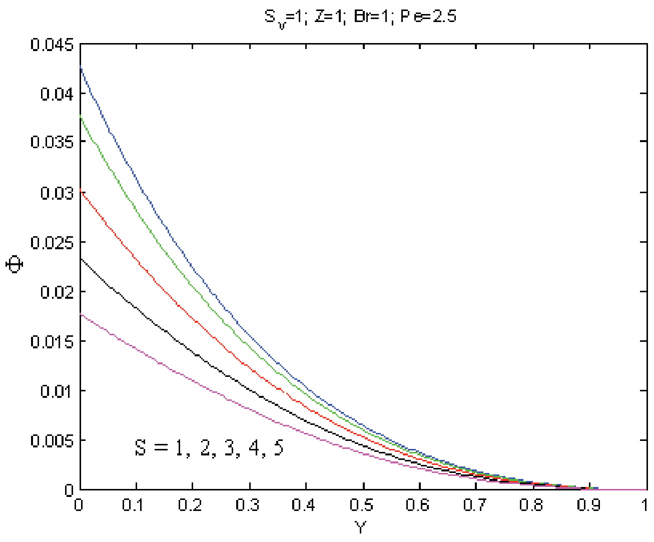

Figure 11. Φ Vs Y for Sv = 1; Z = 1; Br = 1; Pe = 2.5 and S = 1, 2, 3, 4 & 5

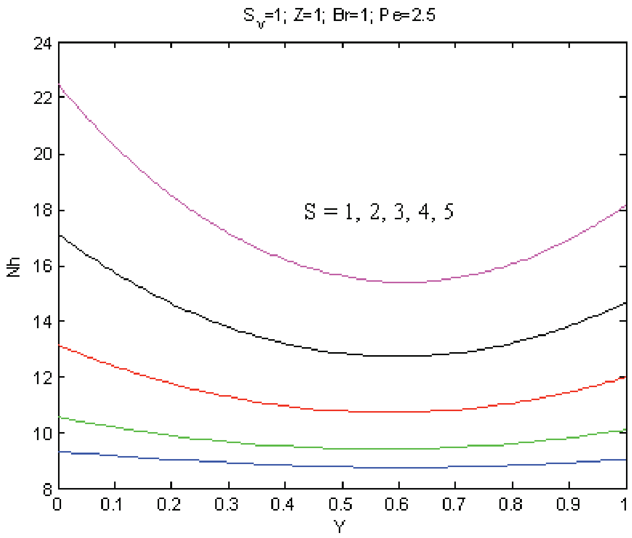

Figure 12. Nh Vs Y for Sv = 1; Z = 1; Br = 1; Pe = 2.5 and S = 1, 2, 3, 4 & 5

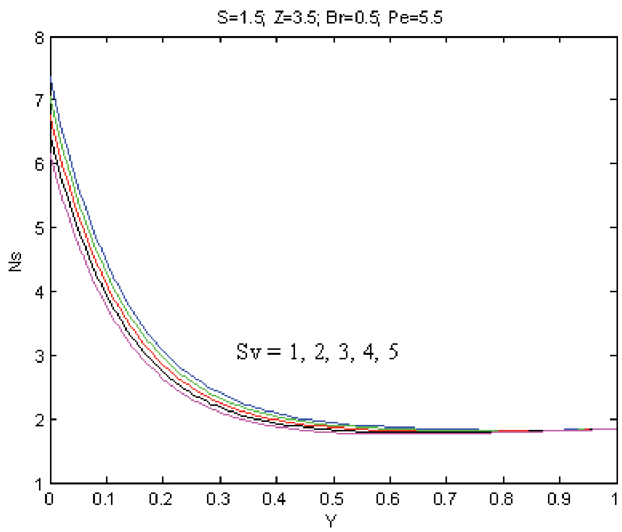

Figure 13. Ns Vs Y for S = 1.5; Z = 3.5; Br = 0.5; Pe = 5.5 and Sv = 1, 2, 3, 4 & 5

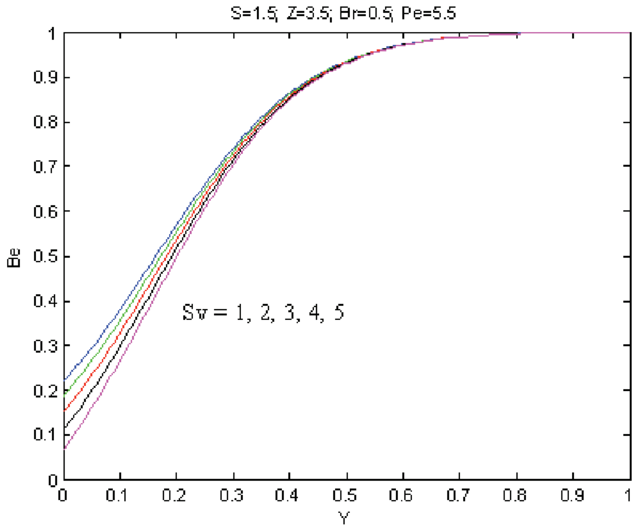

Figure 14. Be Vs Y for S = 1.5; Z = 3.5; Br = 0.5; Pe = 5.5 and Sv = 1, 2, 3, 4 & 5

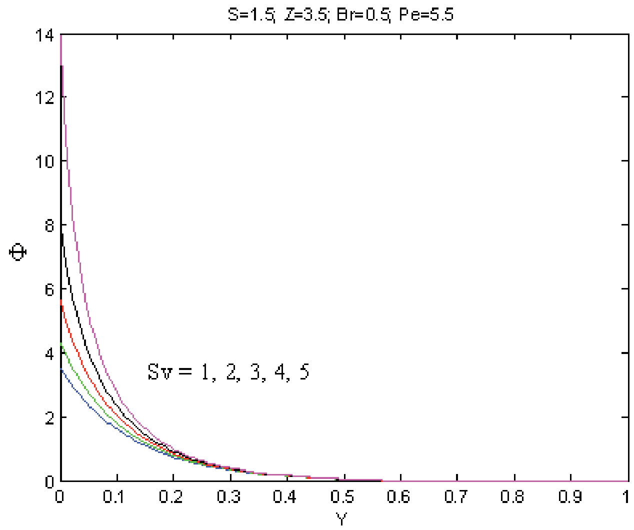

Figure 15. Φ Vs Y for S = 1.5; Z = 3.5; Br = 0.5; Pe = 5.5 and Sv = 1, 2, 3, 4 & 5

CONCLUDING REMARKS

We have considered thermally fully developed electro-osmotically generated flow which is established by a voltage potential gradient along the length of a channel. We have provided an analysis for the entropy generated in a parallel plate microchannel. The boundary condition considered in our case is constant wall heat flux. We have identified five different variables namely, (i) dimensionless joule heating parameter, (ii) dimensionless viscous heating parameter, (iii) relative channel height (iv) Peclet number and (v) Brinkman number from the dimensionless entropy generation number equation. We have analyzed the nature of various results for dimensionless entropy generation number, Bejan number, Irreversibility ratio, entropy generation due to fluid friction, entropy generation due to heat transfer in the axial and transverse direction etc. and discussed the effects of each of those five variables on these parameters.

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