

# Temperature dependence of free-molecular gaseous heat flow in unsealed system open to surrounding ambient

Abhijit Arun Pandhari<sup>1</sup> and Didericus P. H. Hasselman<sup>2</sup>

<sup>1</sup>UBC Materials Engineering, Frank Forward Building, 309-6350 Stores Road, Vancouver, BC, V6T1Z4, Canada, abhijitpandhari@gmail.com

<sup>2</sup>UBC Materials Engineering, Frank Forward Building, 309-6350 Stores Road, Vancouver, BC, V6T1Z4, Canada. hasselmn@vt.edu

## Abstract

Equipment designed for manufacturing or testing of materials at elevated temperatures is often open to and at the pressure of the surrounding ambient. Since, it is an open system, with an increase in temperature the gas phase is expected to expand. This causes reduction in the density of the gas phase within the components or samples contained within the equipment. This reduction in density is linearly proportional to an increase in the temperature. As a consequence, because of its direct dependence on gas density, the rate of gaseous heat flow in the free-molecular regime within these components or samples should exhibit a corresponding decrease with increasing temperature as well. An analysis of this effect conducted by incorporating the effect of temperature on density at constant pressure in the original Knudsen-Kennard formulation, showed that the rate of gaseous heat transfer in the free-molecular regime in an open system at constant pressure is expected to exhibit a reciprocal square root dependence on temperature in contrast to the square root dependence of a sealed system in which the gas density is independent of temperature.

## 1. INTRODUCTION

Many engineering designs in the electronic, nuclear, power generation, manufacturing and other industries, involve the flow of heat across areas of contact between components. Such heat transfer can occur by the combined contributions of direct physical contact, radiation and gaseous conduction. The mechanism of gaseous heat transfer is a function of the Knudsen number ( $Kn$ ),

$$Kn = \lambda/d \quad (1)$$

Where  $\lambda$  is the mean-free-path of the gaseous species and  $d$  is the width of the gap across which the heat is conducted. When  $Kn \ll 1$ , referred to as the “continuum regime”, the rate of heat flow is proportional to the thermal conductivity of the gas, inversely proportional to  $d$  and independent of the density of the gas. As  $Kn$  increases, the heat flow passes through the “temperature-jump” and “transition” regimes. When  $Kn \gg 1$ , the rate of heat flow is independent of  $d$  and directly proportional to the gas density. This regime is referred to as the “free-molecular regime” and is typically observed in micro- and nano- scale components in which the width of the gap is much smaller than the mean free path. This regime is the focus of this communication.

The generally accepted equation for the heat flux,  $H_s$  per unit area and unit time in the free-molecular regime within the gap between parallel surfaces, as originally derived by Knudsen [1], and as presented by Kennard [2] is:

$$H_s = \left( \frac{a}{2-a} \right) \rho \left( \frac{RT}{2\pi} \right)^{1/2} \left( C_v + \frac{1}{2}R \right) (T_2 - T_1) \quad (2)$$

where  $a$  is the thermal accommodation coefficient (for simplicity assumed to be equal for both surfaces),  $\rho$ ,  $T$  and  $C_v$  are the gas density, mean temperature and molar specific heat at constant volume, respectively.  $R$  is the gas constant and  $(T_2 - T_1)$  is the temperature difference between the two surfaces. The use of the subscript,  $s$ , will become clear shortly.

Eqn (2) indicates that  $H_s$  is proportional to  $T^{1/2}$ . This is the direct result of the velocity of the gas atoms or molecules being proportional to  $T^{1/2}$  [2]. The absence of any other temperature dependent term in eqn (2) indicates that in its derivation, the gas density was implicitly assumed to be independent of temperature. For that very reason,  $H_s$  as described by eqn (2) is strictly valid for a gas at constant volume (i.e., “sealed off” from the surrounding ambient), a boundary condition not explicitly stated in the original derivation.

The equipment for the manufacture of materials and measurement of their properties at room and at elevated temperatures generally contains a centrally located temperature-controlled “hot zone”, which contains the samples to be manufactured or measured. This hot zone is thermally insulated by a porous refractory solid, fiber blanket or powder. Usually in the manufacture of materials or the measurement of their properties (especially in air) such as thermal conductivity by the guarded hot plate, “hot wire” and “heated probe” methods [3-5], the hot zone and thermal insulation are not sealed off from the surrounding ambient.

This leads to the inevitable conclusion that at steady-state the gas pressure in the hot zone and in all other components, regardless of their temperature, will be spatially uniform and equal to that of the surrounding ambient. As the result of thermal expansion the gas phase within the hot zone and samples will expand with increasing temperature. In order to maintain constant pressure in an unsealed, open system this increase in volume will be exuded to the surrounding ambient. In turn, this leads to a corresponding decrease in the gas density, (i. e., an effective evacuation) directly proportional to the increase in temperature. For a linear dependence between gas volume and temperature, the gas density,  $\rho$  at temperature,  $T$  can be expressed  $\rho = \rho_{amb} (T_{amb}/T)$ , where  $\rho_{amb}$  and  $T_{amb}$  are the density and temperature of the surrounding ambient, respectively. For values of  $T$  and  $T_{amb}$  equal to 600 K and 300 K, respectively, the gas density at 600 K is one half of the value at 300 K. At  $T$  of 1800 K, a value not uncommon in manufacturing practice, the gas density is reduced to one sixth of its value at 300 K. In contrast, near the temperature of liquid nitrogen of 77 K the gas density will be about four times higher than its value at 300 K.

Because of the small dimensions of the spaces between the particles, gaseous heat flow in nano-sized powders is expected to occur primarily in the free-molecular regime. As the direct result of its dependence on gas density the rate of gaseous heat flow in such powders in an open system at constant ambient pressure should exhibit a corresponding decrease with temperature. The original study of Knudsen [1] and the texts of Kennard [2], Loeb [6], Jeans [7] and Kaganer [8] appear to have focused on sealed systems only, apparently without having recognized that in practice many systems are unsealed and operate at ambient pressure. Experimental [9-20] and modeling studies [21-31] of the effective thermal conductivity of powders or packed beds focused on gaseous heat transfer in the continuum regime not affected by the gas density and therefore applicable to both sealed and open systems.

The purpose of this communication is to present an analysis of this effect with primary emphasis on its temperature dependence, which on dimensional grounds is independent of geometry. Hence, the complexity of the geometry of the open spaces in powder or fibrous insulating materials is avoided. This allowed us to adapt the original analyses of Knudsen and Kennard based on a parallel plate model with sealed ends to the same model but with ends open to the ambient atmosphere as shown in Figure 1. The length of the gap is assumed to be much greater than the width of the gap so that at thermal equilibrium end-effects, if any, can be regarded as negligible.

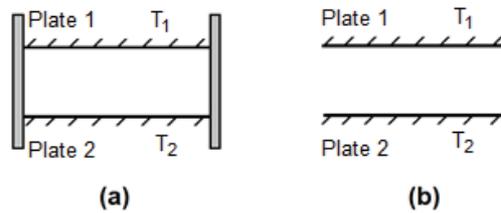


Figure 1. Schematic representation of the systems considered for the analysis of heat flow in the free-molecular regime across a gap between two parallel plates. (a) Sealed gap (b) Open (“unsealed”) gap

**2. ANALYSIS**

In the derivation of the rate of gaseous heat transfer,  $H_s$ , in the free-molecular regime eqn (2), the pressure of the gas between the plates was taken as  $P_{amb}$ , the pressure of the surrounding ambient atmosphere. As presented by Kennard [32], the gas density,  $\rho$ , can be expressed in terms of  $P_{amb}$  and the temperature  $T$ , by eqn (3).

$$\rho = P_{amb}/RT \tag{3}$$

$C_v$  in eqn (2) can be replaced with molar specific heat at constant pressure,  $C_p$ , by;

$$C_v = C_p - R \tag{4}$$

Substitution of eqn (3) and eqn (4) into eqn (2) yields;

$$H_o = P_{amb} \left( \frac{a}{2-a} \right) \left( \frac{1}{2\pi RT} \right)^{1/2} \left( C_p - \frac{1}{2}R \right) (T_2 - T_1) \tag{5}$$

**3. DISCUSSION AND CONCLUSIONS**

Comparison of eqns (2) and (5) shows that in an open system the rate of heat flow exhibits a negative  $T^{-1/2}$  temperature dependence, in contrast to the positive  $T^{1/2}$  for a sealed system. This difference is the direct result of the  $T^{-1}$  dependence of gas density on temperature for the open system. Note that eqns (2) and (5) are continuous at  $T=T_{amb}$  and therefore should be applicable to both elevated and sub-ambient temperatures. Due to the decrease in gas density at  $T > T_{amb}$ ,  $H_s > H_o$ , whereas due to the increase in gas density at  $T < T_{amb}$ ,  $H_s < H_o$ . The expected temperature dependence for both  $H_s$  and  $H_o$  is shown schematically in Figure 2.

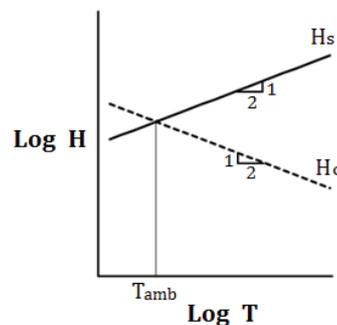


Figure 2. Schematic representation of the effect of temperature on the free-molecular heat transfer in a sealed and an open system

For the purpose of illustration let us consider two numerical examples. For  $T = 600$  K and  $T_{amb} = 300$  K, considered earlier, Eq. (2) yields  $H_s$  at 600 K to be higher by a factor  $\sim 1.4$  than at 300 K. In contrast  $H_o$  at 600 K is  $\sim 0.71$  of the value at 300 K, with the overall result of  $H_o / H_s = \sim 0.5$ . At  $T = 77$  K,  $H_s = \sim 0.5$  of the value at 300 K, whereas  $H_o$  is approximately twice the value at 300 K, yielding  $H_o / H_s = \sim 4$ . These differences between heat fluxes for sealed and open system at both these temperatures are very significant.

Since most of the studies do not specifically target towards gathering the data for both; an open and a closed system, the predicted behavior shown in Figure 2 does not seem to have appeared in the literature. Obtaining such a data may well be regarded as a significant contribution to the heat transfer literature mainly due to its practical significance.

The above results lead us to conclude that estimates or interpretations of experimental data for gaseous heat fluxes in systems open to the surrounding atmosphere, based on eqn (2), strictly valid for a sealed system, could be in considerable error. This will be particularly so for temperatures which differ significantly from ambient. The results of this study indicate that in engineering practices and in fundamental studies of gaseous heat transfer it is imperative to explicitly define whether the system is sealed off or open to the ambient atmosphere.

The effect that has been identified in this work may have significant implications in the engineering design, specifically the “thermal management,” of any system which operates at elevated temperatures. As an example, solid-state electronic components subject to a given heat flux should be sealed in order to keep a peak temperature to a minimum. However, systems which require optimum thermal insulation should be open as much as possible in order to minimize the energy consumption.

#### ACKNOWLEDGEMENTS

The authors would like to thank Amber Perreca for proofreading the paper. The authors would like to thank the University of British Columbia, Canada, for financial support.

#### REFERENCES

- [1] M. Knudsen, The Molecular Heat Conduction of Gases and Accommodation Coefficients, *Ann. Phys.*, 1911, 4 (4), 593-656.
- [2] E.H. Kennard, The Kinetic Theory of Gases: With an Introduction to Statistical Mechanics, McGraw-Hill Publishing Co. Ltd., London, 1934, 315-318.
- [3] ASTM D5334-08, Standard Test Method for Determination of Thermal Conductivity of Soil and Soft Rock by Thermal Needle Probe Procedure, *Annual Book of ASTM Standards* 4.08, West Conshohocken (PA), 2008.
- [4] ASTM C1113/1113M-09, Standard Test Method for thermal conductivity of refractories by hot wire (Platinum Resistance Thermometer Technique), *Annual Book of ASTM Standards* 15.01, West Conshohocken (PA), 2009.
- [5] ASTM C177-10, Standard Test Method for Steady-State Measurements and Thermal Transmission Properties by Means of the Guarded-Hot-Plate Apparatus, *Annual Book of ASTM Standards* 4.06, West Conshohocken (PA), 2010.
- [6] L.B. Loeb, The Kinetic Theory of Gases (Book Review), McGraw-Hill Book Company Inc., New-York, 1927, 283-295.
- [7] J.H. Jeans, An Introduction to the Kinetic Theory of Gases, Cambridge University Press, London, 1940, 190-194.
- [8] Kaganer M. G., Thermal Insulation in Cryogenic Engineering, Israel Program for Scientific Translations Ltd, *Jerusalem*, 1969, 152–166.
- [9] Kunii D. and Smith J.M., Thermal Conductivity of Porous Rocks Filled with Stagnant Fluid, *Soc. Petrol. Eng. J.*, 1960, 1 (1), 37-42.
- [10] Swift D.L., The Thermal Conductivity of Spherical Metal Powders Including the Effect of an Oxide Coating, *Int. J. Heat Mass Tran.*, 1966, 9 (10), 1061-1074.

- [11] Godbee H.W. and Ziegler W. T., Thermal Conductivities of MgO, Al<sub>2</sub>O<sub>3</sub>, and ZrO<sub>2</sub> Powders to 850° C. I. Experimental, *J. Appl. Phys.*, 1966, 37 (1), 40-55.
- [12] Luikov A.V., Shashkov A. G., Vasiliev L. L. and Fraiman Yu. E., Thermal Conductivity of Porous Systems, *Int. J. Heat Mass Tran.*, 1966, 11 (2), 117-140.
- [13] McCoy B.J. and Cha C. Y., Transport Phenomena in the Rarefied Gas Transition Regime, *Chem. Eng. Sci.*, 1974, 29 (2), 381-388.
- [14] Elsari M. and Hughes R., Axial Effective Thermal Conductivities of Packed Beds, *Appl. Therm. Eng.*, 1980, 22 (18) 1969-1980.
- [15] Shonnard D.R. and Whitaker S., The Effective Thermal Conductivity for a Point-Contact Porous Medium: an Experimental Study, *Int. J. Heat Mass Tran.*, 1989, 32 (3), 503-512.
- [16] Hall R.O.A., Martin D.G. and Mortimer M.J., The Thermal Conductivity of UO<sub>2</sub> Sphere-Packed Beds, *J. Nucl. Mater.*, 1990, 173 (2), 130-141.
- [17] Caps R., Heinemann U. and Fricke J., Thermal Conductivity of Polyimide Foams, *Int. J. Heat Mass Tran.*, 1997, 40 (2), 269-280.
- [18] Muller S.G., Fricke J., Hofmann D., Horn R., Nilsson O. and Rexer B., Experimental and Theoretical Analysis of the Thermal Conductivity of SiC Powder as Source Material for SiC Bulk Growth, *Mater. Sci. Forum*, 2000, 338-342, 43-46.
- [19] Caps R. and Fricke J., Thermal Conductivity of Opacified Powder Filler Materials for Vacuum Insulations, *Int. J. Thermophys.*, 2000, 21 (2), 445-452.
- [20] Slavin A.J., Londry F. A. and Harrison J., A New Model for the Effective Thermal Conductivity of Packed Beds of Solid Spheroids: Alumina in Helium between 100 and 500°C, *Int. J. Heat Mass Tran.*, 2000, 43 (12), 2059-2073.
- [21] Reim M., Korner W., Manara J., Korder S., Arduini-Schuster M., Ebert H. P. and J. Fricke, Silica Aerogel Granulate Material for Thermal Insulation and Daylighting, *Sol. Energy*, 2005, 79, 131-139.
- [22] Gopalarathnam C.D., Hoelscher H.E. and Laddha G.S., Effective Thermal Conductivity in Packed Beds, *ALChE J.*, 1961, 7 (2), 249-253.
- [23] Argento C. and Bouvard D., Modeling the Effective Thermal Conductivity of Random Packing of Spheres through Densification, *Int. J. Heat Mass Tran.*, 1996, 39 (7), 1343-1350.
- [24] Hayashi S., Kubota K., Masaki H., Shibata Y. and Takahashi K., A theoretical Model for the Estimation of the effective Thermal Conductivity of Packed Bed of Fine Particles, *Chem. Eng. J.*, 1987, 35 (1), 51-60.
- [25] Pons M. and Dantzer P., Effective Thermal Conductivity in Hydride Packed Beds I. Study of Basic Mechanisms with help of the Bauer Schlunder Model, *J. Less-Common Met.*, 1991, 172-174, 1147-1156.
- [26] Starck C. and Fricke J., Improved Heat-Transfer Models for Fibrous Insulations, *Int. J. Heat Mass Tran.*, 1993, 36 (3), 617-625.
- [27] Lund K.O., Nguyen H., Lord S.M. and Thompson S., Numerical Correlation for Thermal Conduction in Packed Beds, *Can. J. Chem. Eng.*, 1999, 77 (4), 769-774.
- [28] Siu W.W.M. and Lee S.H.K., Effective Conductivity Computation of a Packed Bed using Constriction Resistance and Contact Angle Effects, *Int. J. Heat Mass Tran.*, 2000, 43 (21), 3917-3924.
- [29] Bahrami M., Yovanovich M.M. and Culham J.R., Effective Thermal Conductivity of Rough Spherical Packed Beds, *Int. J. Heat Mass Tran.*, 2006, 49 (19-20), 3691-3701.
- [30] Spagnol S., Lartigue B., Trombe A. and Gibiat V., Modeling of Thermal Conduction in Granular Silica Aerogels, *J. Sol-Gel Sci. Techn.*, 2008, 48 (1-2), 40-46.
- [31] Bhatt H., Donaldson K.Y. and Hasselman D.P.H., Role of Interfacial Carbon Layer in the Thermal Diffusivity/Conductivity of Silicon Carbide Fiber-Reinforced Reaction Bonded Silicon Nitride Matrix Composites, *J. Am. Ceram. Soc.*, 1992, 75(2), 334-340.

