Dual solutions for the boundary layer flow of a nanofluid over a moving surface

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Abstract
An analysis of heat and mass transfer for boundary layer forced convective flow of a nanofluid past a moving flat surface parallel to a moving stream is presented. The similarity solutions for the problem are obtained and the reduced ordinary differential equations are solved numerically. To support the validity of the numerical results, comparison is made with known results from the open literature for some particular cases of the present study. When the surface and the fluid move in the opposite directions, dual solutions exist. Numerical results for friction factor, surface heat transfer rate and mass transfer rate have been presented for parametric variations of the Brownian motion parameter Nb, thermophoresis parameter Nt and Lewis number Le. The dependency of the friction factor, surface heat transfer rate (Nusselt number) and mass transfer rate (Sherwood number) on these parameters has been discussed.

Key words: Forced convective flow, moving surface, nanofluid, dual solutions.

NOMENCLATURE

C	concentration
C_p	Specific heat at constant pressure
C_{fx}	Local friction factor
D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
f	dimensionless stream function
k_m	effective thermal conductivity of the porous medium
Le	Lewis number
Nb	Brownian motion parameter
Nt	thermophoresis parameter
Nu	Nusselt number
q”	wall heat flux
Re	Reynolds number
T	temperature
T_W	wall temperature of the vertical plate
T_∞	ambient temperature
U	reference velocity
u, v	Darcy velocity components
(x, y)	Cartesian coordinates

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Greek Symbols:

αm	thermal diffusivity of porous medium
η	dimensionless distance
θ	dimensionless temperature
μ	viscosity of fluid
ρf	fluid density
ρp	nano-particle mass density
$(\rho c)f$	heat capacity of the fluid
$(\rho c)m$	effective heat capacity of porous medium
$(\rho c)p$	effective heat capacity of nano-particle material
τ	ratio of heat capacity of particles to heat capacity of fluid
ϕ	nano-particle volume fraction
ϕ_w	nano-particle volume fraction at the wall of the vertical plate
ϕ_∞	ambient nano-particle volume fraction

Subscript

w	Wall conditions
∞	Ambient temperature
x	Differentiation with respect to x

Superscript

Differentiation with respect to η

1. INTRODUCTION

The forced convection over a flat plate/sheet has been widely studied over the past few decades. Earlier investigators were mainly interested to find the similarity solutions for the boundary layer flow problems. In Fluid mechanics, the problem of viscous boundary-layer flow on a moving or fixed flat plate is a classical problem. Flow and heat transfer of a viscous fluid over a moving surface has many important applications in the modern industry viz. polymer industry, glass fiber drawing, crystal growing, plastic extrusion, continuous casting, etc.(Magyari and Keller [1]).

The boundary layer flow on a flat plate was investigated by different researchers viz. Blasius [2], Howarth [3], Abussita [4],Wang [5] etc. For the boundary layer flow on a moving flat plate in a quiescent fluid, Sakiadis [6] obtained the same equations as obtained by Blasius [2] with different boundary conditions. By introducing composite velocity, Afzal et al. [7] combined Blasius and Sakiadis problems successfully and obtained a single set of equations. Recently, the Blasius and Sakiadis problems were extended respectively by Batallar [8] and Cortell [9] by studying the effects of radiation on the boundary layer. Considering the suction/blowing effect on the boundary, a new dimension is added to the above mentioned study by Ishak [10].

Siekman [11], Klemp and Acrivos [12], Abdulhafez [13], Chappidi and Gunnerson [14], Hussaini et al. [15], Lin and Haung [16] and Sparrow and Abraham [17] reported the flow, heat transfer characteristics for moving wall laminar boundary layer problems.

Cortell [18] extended the work of Afzal et al. [7] for constant as well as prescribed power-law surface temperature. However, the existence of dual solutions was not discussed in that study. Recently, Ishak et al. [19] showed that dual solutions exist when the velocity ratio exceeds unity, i.e. the sheet moves in opposite direction to the free stream. The effects of suction and injection on the flow and thermal fields for constant surface temperature were also reported in that study. But in his study the prescribed surface temperature was disregarded. Of late, Mukhopadhyay [20] investigated the case of prescribed surface temperature of the second degree and reported the existence of dual solutions.

Many researchers are studying the convective heat transfer in nanofluids. The nanofluids have many industrial applications since materials of nanometer size have unique physical and chemical properties. Nanofluids are solid-liquid composite materials consisting of solid nanoparticles or nanofibers with sizes typically of 1-100 nm suspended in liquid. Nanofluids have attracted great interest recently because of reports of greatly enhanced thermal properties. It is reported that a small amount (<1% volume fraction) of Cu nanoparticles or carbon nanotubes dispersed in ethylene glycol or oil will increase the inherently poor thermal conductivity of the liquid by 40% and 150%, respectively [23,24]. Conventional particle-liquid suspensions require high concentrations (>10%) of particles to achieve such enhancement. However, problems of rheology and stability are amplified at high concentrations, precluding the widespread use of conventional slurries as heat transfer fluids. In some cases, the observed enhancement in thermal conductivity of nanofluids is orders of magnitude larger than predicted by well-established theories. Nanofluids display a strong temperature dependence of the thermal conductivity [25] and a three-fold higher critical heat flux compared with the base fluids [26,27]. If confirmed and found consistent, they would make nanofluids promising for applications in thermal management. Furthermore, suspensions of metal nanoparticles are also being developed for other purposes, such as medical applications including cancer therapy. The interdisciplinary nature of nanofluid research presents a great opportunity for exploration and discovery at the frontiers of nanotechnology.

The present work has been undertaken in order to study the heat transfer for boundary layer forced convective flow of a nanofluid past a moving flat surface parallel to a moving stream. The similarity solutions for the problem are obtained and the reduced ordinary differential equations are solved numerically. We considered more general power-law surface temperature and concentration at the boundary.

2. FORMULATION OF THE PROBLEM

We consider a forced convective, two-dimensional steady laminar boundary-layer flow of a nanofluid over a flat surface moving with constant velocity U_w in the same or opposite direction to the free stream U_∞ (directed towards the positive x-direction). The x-axis extends parallel to the surface, while y-axis extends upwards, normal to the surface. Figure 1 shows the flow model and coordinate system. The governing equations for boundary layer flows and heat transfer are written as

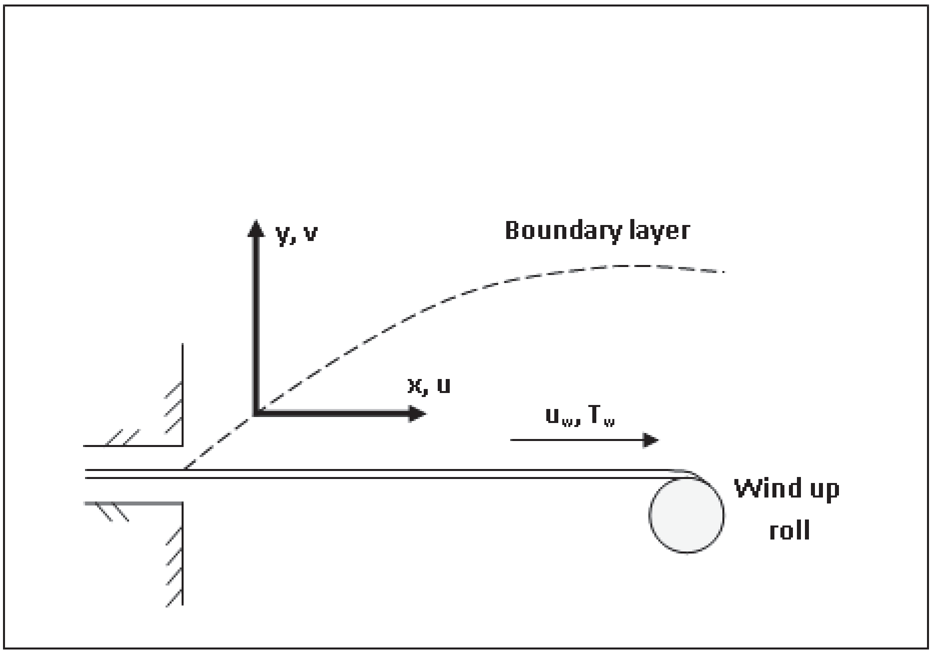

Figure 1 Flow Model and Coordinate System

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\mu u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\alpha_m\nabla^2 T+\tau\left(D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right) \tag{3}$$

$$u\frac{\partial C}{\partial x}+\upsilon\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

The appropriate boundary conditions for the problem are given by

$$u=U_w, \upsilon=0, T=T_w, \ C = C_w \text{ at } y = 0, \tag{5}$$

$$u\to U_\infty, T\to T_\infty, C\to C_\infty, \text{ as } y \to \infty. \tag{6}$$

Here $T_w = T_\infty + Ax^n$ is the prescribed surface temperature and $C_w = C_\infty + Bx^n$ is the prescribed concentration and is the power law exponent.

2.1Similarity analysis and solution procedure

With the help of a composite velocity $U = U_w + U_\infty$ we now introduce the following dimensionless variables

$$\eta = y\sqrt{\frac{U}{2\nu x}},\ u=Uf'(\eta),\ \upsilon=U\ \frac{\eta f'(\eta)-f(\eta)}{\sqrt{2\,\mathrm{Re}_x}} \tag{7}$$

$$\theta=\frac{T-T_\infty}{T_w-T_\infty}, \varphi=\frac{C-C_\infty}{C_w-C_\infty} \tag{8}$$

$\mathrm{Re}_x = \dfrac{Ux}{\nu}$ is the local Reynolds number.

Using the relations (7) and (8) in the governing boundary layer equations (2-4), we get the following equations:

$$f''' + ff'' = 0, \tag{9}$$

$$\theta'' + \Pr(f\theta' - 2\mathrm{n}f'\theta) + N_B\theta'\phi' + N_t(\theta')^2 = 0, \tag{10}$$

$$\varphi'' + Sc(f\varphi' - 2\mathrm{n}f'\varphi) + \frac{N_t}{N_B}\theta'' = 0. \tag{11}$$

The transformed boundary conditions then become

$$f' = 1-r, f = 0, \theta = 1, \varphi = 1 \text{ at } \eta = 0\ ,$$

and

$$f' = r, \theta = 0, \phi = 0 \text{ as } \eta \to \infty \tag{12}$$

where $r = \dfrac{U_\infty}{U}$ is the velocity ratio parameter.

Equations (9-11) along with boundary conditions (12) were solved numerically by shooting method (Mukhopadhyay et al. [21], Bhattacharyya et al. [22]). The details about the directions of the moving wall and the free stream can be found from Mukhopadhyay [20].

3. RESULTS AND DISCUSSIONS

In order to get a clear insight of the physical problem, numerical computations have been carried out using shooting method for various values of different parameters such as velocity ratio parameter (r), power-law exponent (n) and Prandtl number (Pr), Scmidt number (Sc), Brownian motion parameter (Nb) and thermophoresis parameter (Nt) encountered in this problem.

In order to assess the accuracy of the method, the numerical results obtained in this study are compared with those of Ishak et al. [19] for Nb = Nt = 0 and variable $r(r > 1)$ and presented in Table 1. Our results are in excellent agreement with literature data. This would suggest that our results are highly accurate. For the sake of brevity, most of the numerical results of this study are presented for n = 0.5, r = 1.1.

Tables 2-4 display results for wall values for the gradients of temperature and concentration functions which are proportional to the Nusselt number and Sherwood number, respectively. Table 2 shows that as Nb increases, the surface mass transfer rates increase whereas the surface heat transfer rate decreases. Nb is the Brownian motion parameter. Brownian motion decelerates the flow in the nanofluid boundary layer. Brownian diffusion promotes heat conduction. The nanoparticles increase the surface area for heat transfer. Nanofluid is a two phase fluid where the nanoparticles move randomly and increase the energy exchange rates. Brownian motion reduces nanoparticle diffusion. Figures 2 and 3 show that as Nb increases, temperature increases and concentration decreases within the boundary layer.

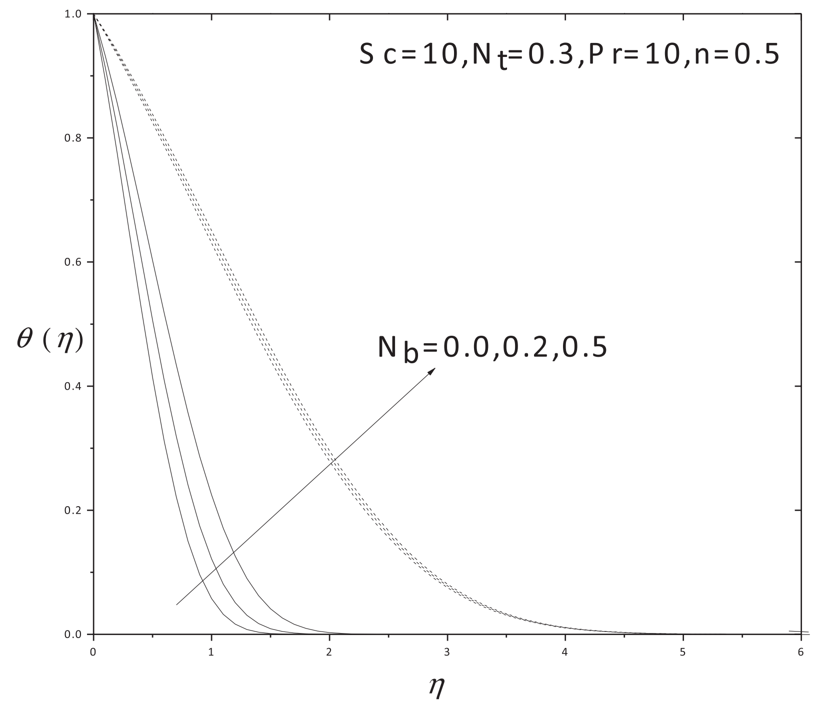

Figure 2 Temperature distribution for variable values of Brownian motion parameter Nb.

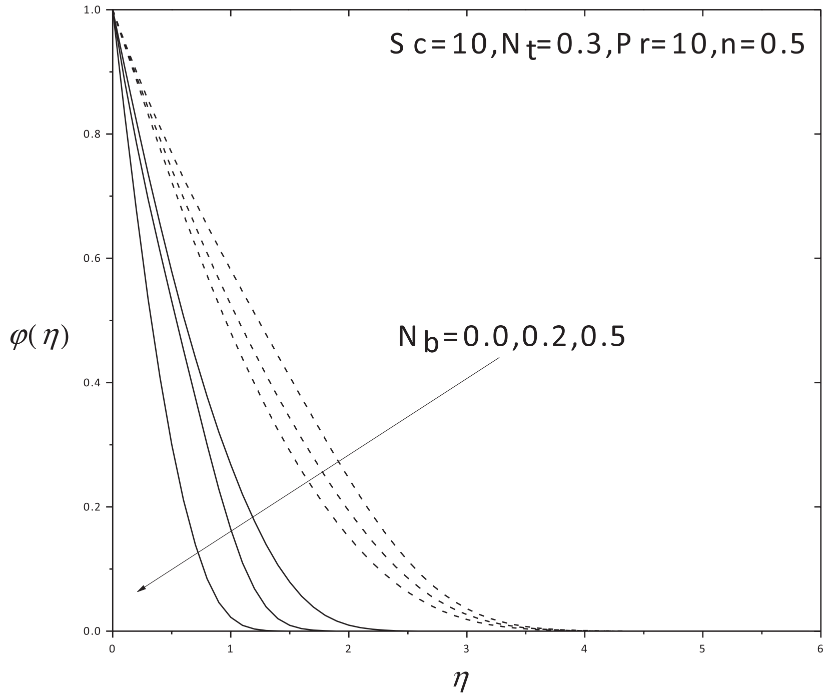

Figure 3 Concentration distribution for variable values of Brownian motion parameter Nb

Table-1: Values of velocity gradient $f''(0)$ for non-porous flat surface

r	$f''(0)$			
	Ishak et al. [19]		Present study	
	Upper branch	Lower branch	Upper branch	Lower branch
1.1	0.533708	0.001493	0.533707	0.001491
1.2	0.583178	0.016171	0.583176	0.016172
1.3	0.613646	0.051941	0.613645	0.051940
1.4	0.616140	0.117886	0.616142	0.117885
1.5	0.565821	0.241872	0.565823	0.241874

Table -2: Values of , $f^{//}(0)$, - $\theta^{/}(0)$, - $\varphi^{/}(0)$ for variable N_b when N_t=0.3, n=0.5, Sc=10, Pr=10

N_b	$f^{//}(0)$		$-\theta^{/}(0)$		$-\varphi^{/}(0)$	
	Upper branch	Lower branch	Upper branch	Lower branch	Upper branch	Lower branch
0.0	0.533707	0.001491	1.457979	0.909908	-5.169398	-1.253891
0.1			1.396806	0.869207	1.047841	0.881666
0.2			1.330423	0.825062	1.428545	1.011311
0.3			1.266499	0.782562	1.553021	1.052862
0.4			1.204970	0.741746	1.613487	1.072464
0.5			1.145809	0.702389	1.648375	1.083226

Table -3: Values of $f^{//}(0)$, - $\theta^{/}(0)$, - $\varphi^{/}(0)$ for variable N_t when N_b=0.3, n = 0.5, Sc=10, Pr=10

N_t	$f^{//}(0)$		$-\theta^{/}(0)$		$-\varphi^{/}(0)$	
	Upper branch	Lower branch	Upper branch	Lower branch	Upper branch	Lower branch
0.0	0.533707	0.001491	1.421437	0.885430	1.641448	1.032086
0.1			1.367715	0.849738	1.584473	1.021343
0.2			1.316098	0.815501	1.555633	1.028707
0.3			1.266499	0.782562	1.553021	1.052862
0.4			1.218879	0.751151	1.574679	1.092509
0.5			1.173176	0.720987	1.618820	1.146365

Table 3 shows that as Nt increases, the heat transfer rate increases/decreases for the lower branch/upper branch respectively and mass transfer rate decreases/increases for upper and lower branches respectively. Thermophoresis parameter, Nt appears in the thermal and concentration boundary layer equations. As we note, it is coupled with temperature function and plays a strong role in determining the diffusion of heat and nanoparticle concentration in the boundary layer. From Figures 4 and 5, we note that the temperature and nanoparticle concentration are elevated as Nt increases.

As Le increases, the heat transfer rate decreases whereas the mass transfer rate increases as shown by Table 4. Le represents the ratio of molecular thermal diffusivity to mass diffusivity (Sh and Pr). As Le increases, the thermal diffusivity is more pronounced than the mass diffusivity. Since large values of Le make the molecular diffusivity smaller, concentration decreases as Le increases. Figure 6 shows that as Le increases, the concentration within the boundary layer decreases and the concentration boundary layer thicknesses decreases.

The influence of nanoparticles on forced convection is modeled by accounting for Brownian motion and thermophoresis as well as non-isothermal boundary conditions. The thickness of the boundary layer for the mass fraction is smaller than the thermal boundary layer thickness for Large values of Lewis number, Le. The contribution of N_t to heat and mass transfer does not depend on the value of Le. The Brownian motion and thermophoresis of nanoparticles increases the effective thermal conductivity of the nanofluid. Both Brownian diffusion and thermophoresis give rise to cross diffusion terms that are similar to the familiar Soret and Dufour cross diffusion terms that arise with a binary fluid discussed by Lakshmi Narayana et al. [28].

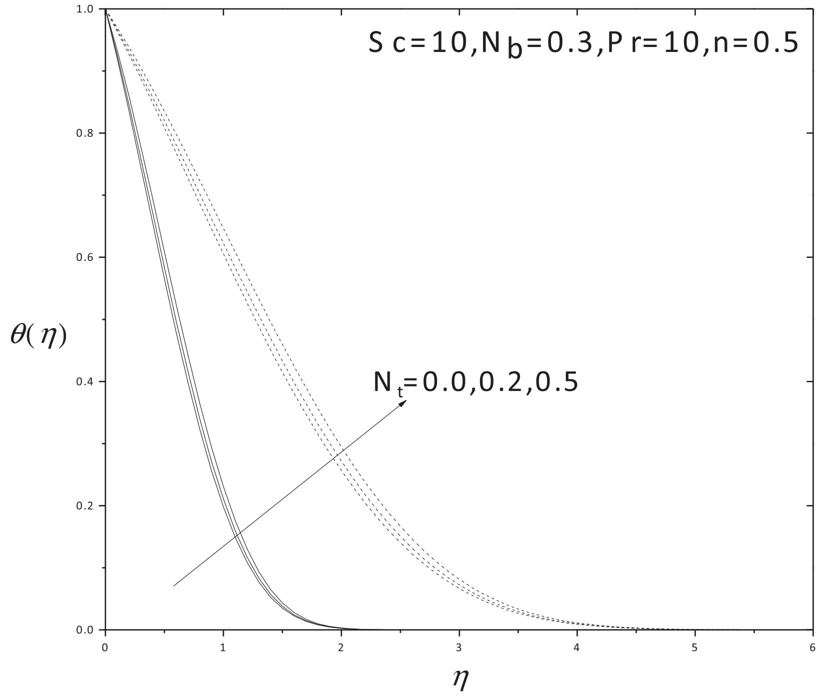

Figure 4 Temperature distribution for variable values of thermophoresis parameter Nt.

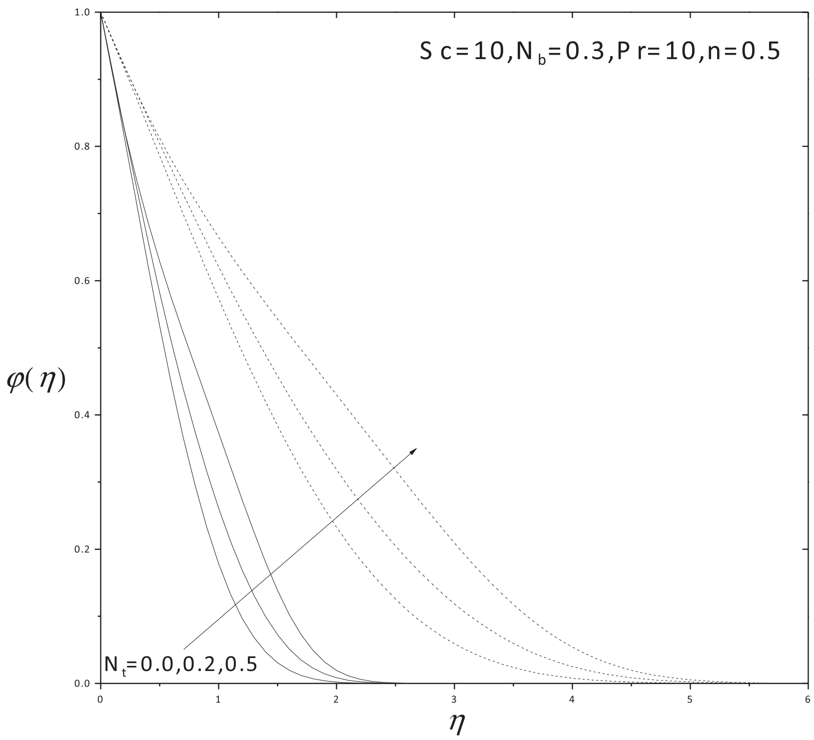

Figure 5 Concentration distribution for variable values of thermophoresis parameter Nt.

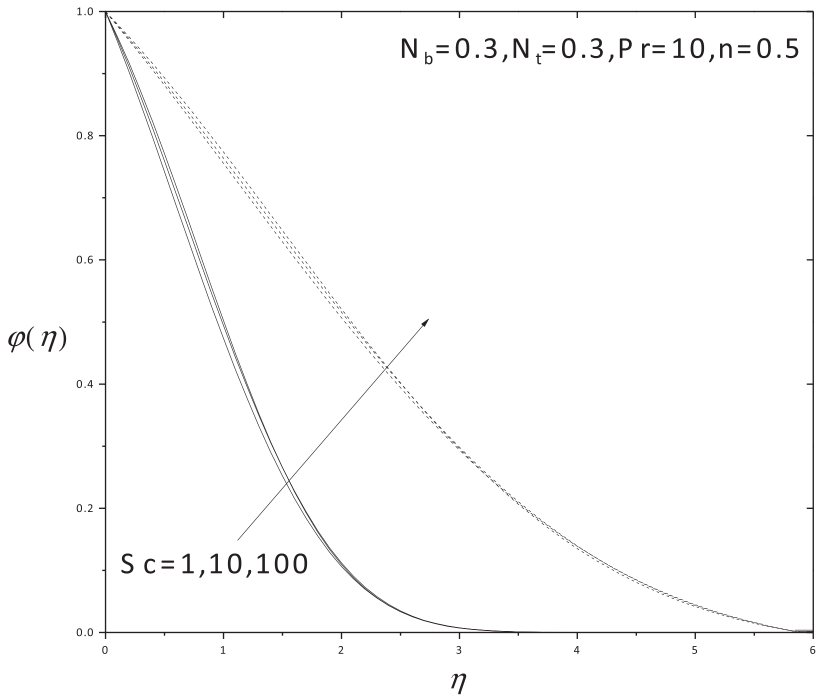

Figure 6. Concentration distribution for variable values of Schmidt number Sc.

Table -4: Values of $\theta'(0)$, - $\varphi'(0)$ for variable Sc when N_b=0.3, N_t=0.3, n = 0.5, Pr=10

Sc	$-\theta'(0)$		$-\varphi'(0)$	
	Upper branch	Lower branch	Upper branch	Lower branch
1	1.336763	0.872467	0.791372	0.248509
10	1.266499	0.782562	1.553021	1.052862
100	1.166775	0.747938	2.812732	1.180894

Figure 7 & 8 exhibit the nature of thermal and concentration profiles for variable power-law index n when the surface temperature and concentration vary directly with n (i.e. for n>0). Temperature [Figure 7] as well as concentration [Figure 8] in the boundary layer decreases with increasing values of n. In this case, the thermal and solute boundary layer decrease. On the other hand, when the sheet temperature and concentration vary inversely with n (i.e. for n<0), temperature [Figure 9] as well as concentration [Figure 10] increases with increasing absolute values of n (for n<0).

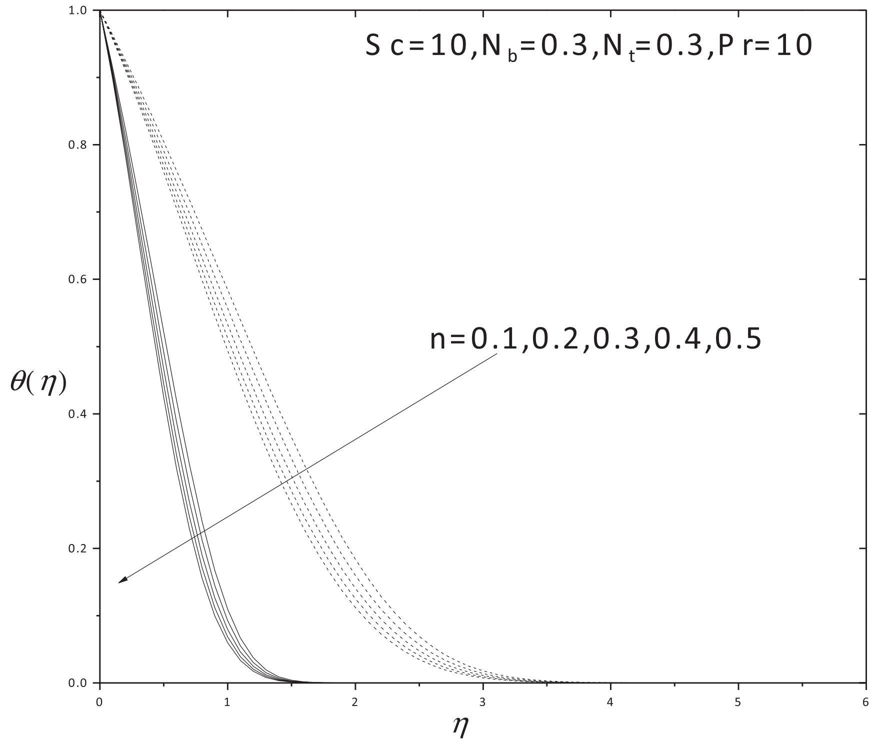

Figure 7. Temperature distribution for variable positive values of power law index n.

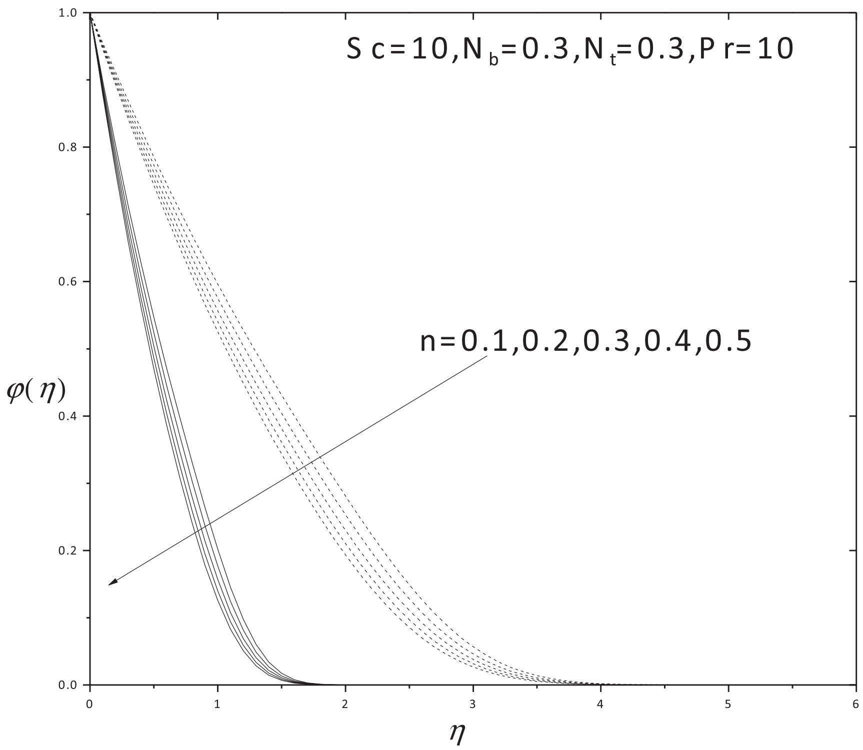

Figure 8. Concentration distribution for variable positive values of power law index n

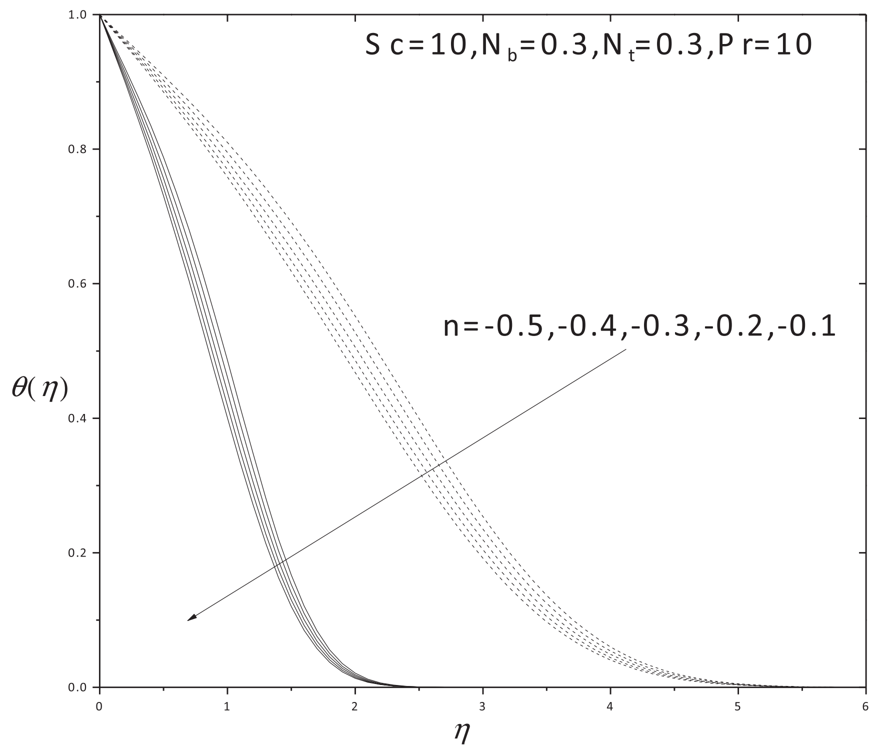

Figure 9. Temperature distribution for variable negative values of power law index n.

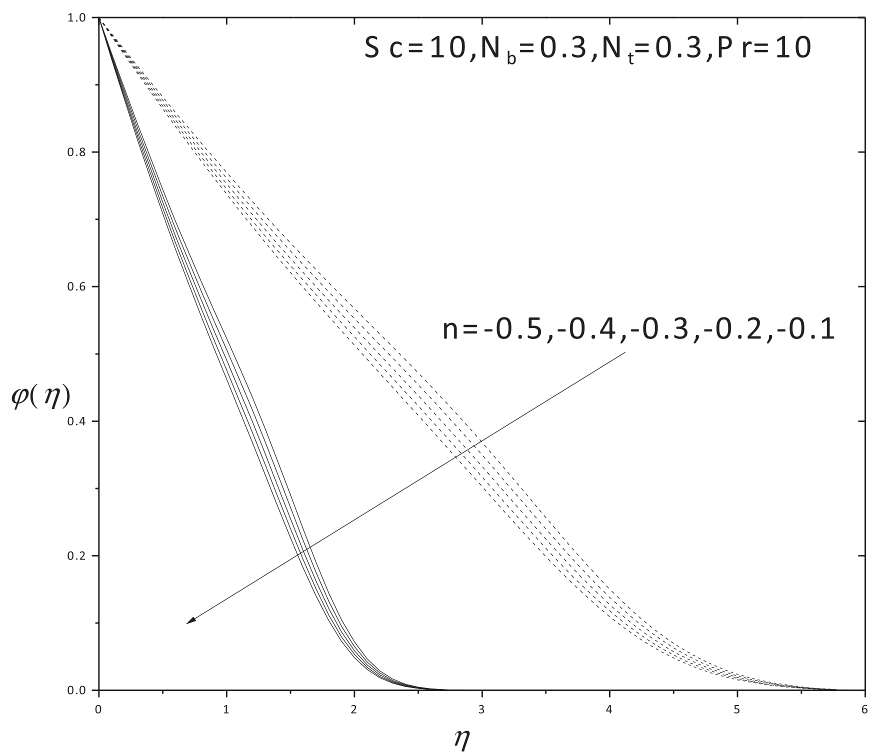

Figure 10. Concentration distribution for variable negative values of power law index n .

4. CONCLUSIONS:

In this paper, we presented a nanofluid boundary layer analysis for the heat transfer characteristics for a moving surface parallel to a free stream. Dual solutions are obtained when the plate and the fluid move in the opposite direction. Numerical results for surface heat transfer rate and mass transfer rate have been presented for parametric variations of the Brownian motion parameter Nb, thermophoresis parameter Nt and Lewis number Le. The results indicate that as Nt increases, the heat transfer rate (Nusselt number) and mass transfer rate (Sherwood number) decrease for the upper branch and increase for the lower branch. As Nb increases, the surface mass transfer rates increase whereas the surface heat transfer rate decreases. As Le increases, the heat transfer rate decreases whereas the mass transfer rate increases.

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