Using Native Inhomogeneity of Heterostructure to Decrease Dimensions of Planar Field-Effect Transistors

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In this paper we consider an approach to decrease dimensions of planar field-effect transistors in a semiconductor heterostructure. It has been formulated a recommendation to use inhomogeneity of heterostructure to increase the effect.

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1. INTRODUCTION

To decrease dimensions of elements of integrated circuits (IC) (such as *p*-*n*-junctions and transistors) it has been used some approaches [1–6]. One of them is formation of inhomogenous distribution of temperature (during laser or microwave annealing) [4, 7, 8]. It could be also interesting of inhomogenous distribution of defects [9]. Another way to decrease the dimensions of the devices (discrete devices and elements of integrated circuits) is optimization of technological process. Framework the approach we consider a possibility to decrease dimensions (increasing of density of elements of IC and decreasing of depth of devices) of planar field-effect transistors.

2. STATEMENT OF THE PROBLEM

Let us consider a heterostructure (H). The H consist of a substrate (S) with known type of conductivity (*n* or *p*) and two epitaxial layers (EL) (see Fig. 1). In inserted parts of the EL one or two dopants (depends on quantity of inserted parts) have been infused. Further are have been a layer of oxide and contacts. Profile of side elevation drawing of the obtained structure is presented in Fig. 2. The Fig. 3 and 4 are illustrated relations between initial distributions of dopant and structure of H in neighborhood of last contacts. It has been recently shown (see, for example, [10, 11]), that optimization of annealing time gives us possibility to manufacturing a *p*-*n*-junction with higher sharpness and higher homogeneity of dopant distribution near appropriate interface between layers of H. Main aim of the paper is the optimization of annealing time to manufacturing more compact field-effect transistors.

3. METHOD OF SOLUTION

To solve our aims let us determine spatiotemporal distribution of dopant concentration. The distribution we determine by solving the second Fick's law [1–3]

$$\frac{\partial C(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_C\frac{\partial C(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_C\frac{\partial C(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D_C\frac{\partial C(x,y,z,t)}{\partial z}\right] \qquad (1)$$

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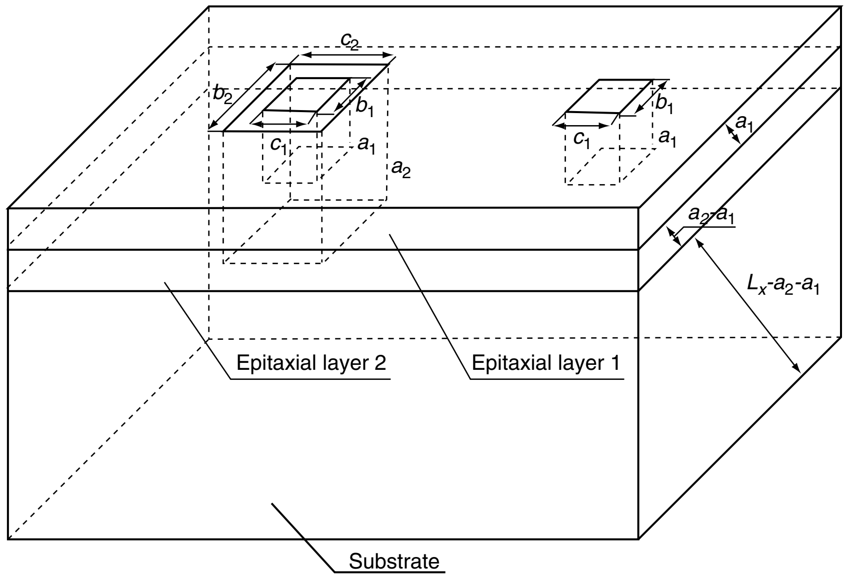

Figure1. Heterostructure with two epitaxial layers, which include into itself two or three inserted parts. The figure also illustrates initial (before starting of annealing) distributions of dopant.

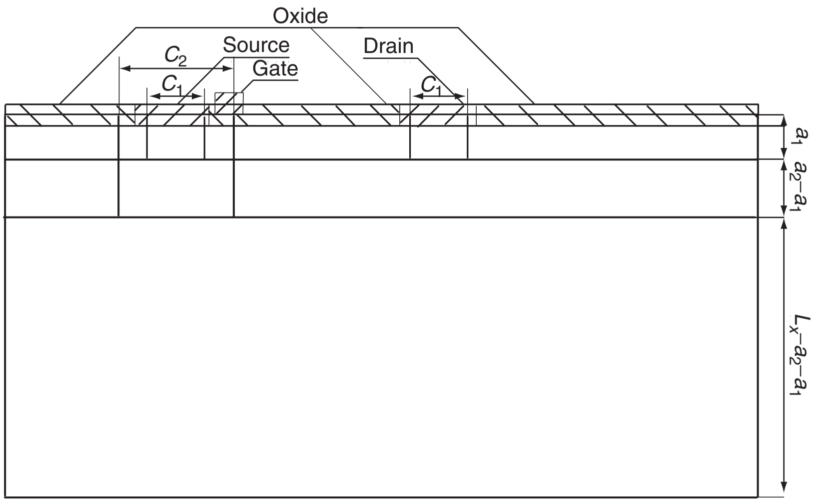

Figure 2. Profile of side elevation drawing of the obtained heterostructure.

$$\left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=0}=0,\left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{y=0}=0,\left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{x=L_y}$$
$$=0,\left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{z=0}=0,\left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{x=L_z}=0, C(x,y,z,0)=fc(x,y,z). \tag{2}$$

Here $C(x,y,z,t)$ is the spatiotemporal distribution of dopant concentration; T is the annealing temperature; D_C is the diffusion coefficient of dopant. Value of dopant diffusion coefficient depends on properties of materials of layers in H, on rate of heating and cooling of H and on spatiotemporal distribution of dopant concentration.

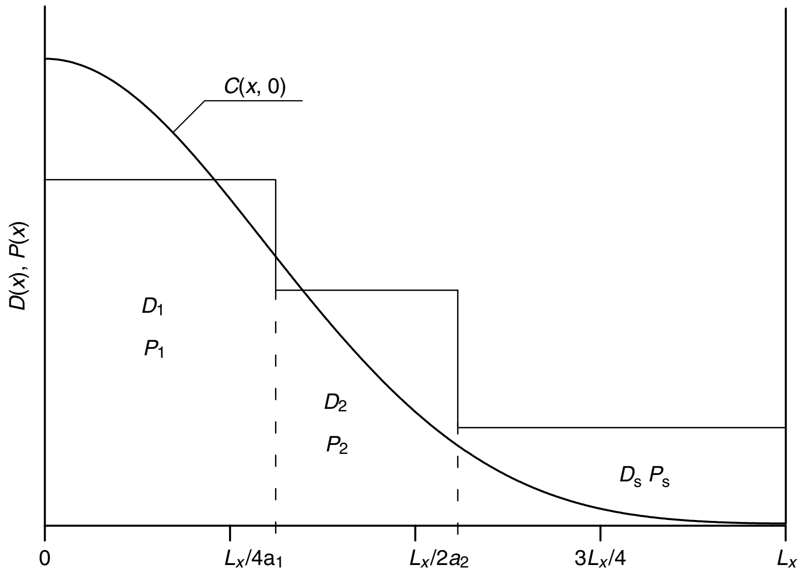

Figure 3. Initial distribution of dopant near drain.

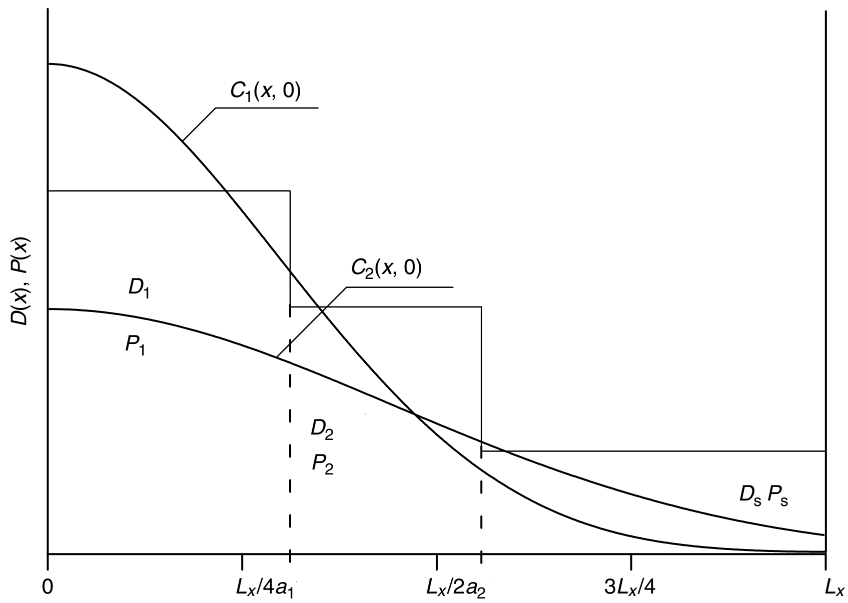

Figure 4. Initial distribution of dopant near source.

It has been shown in [3], that in high-doped materials interaction between dopant atoms and point defects increases. If the point defects have nonzero charge γe with e an elementary charge, then the interaction leads to concentrational dependence of the diffusion coefficient. The dependence can be approximated by a power law

$$D_C = D_L(x,y,z,T)\left[1+\xi\frac{C^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\right]. \tag{3}$$

Here $P\ (x,y,z,T)$ is the limit of solubility of dopant in H; $D_L\ (x,y,z,T)$ is the diffusion coefficients for linear diffusion (the diffusion independent on dopant concentration, the diffusion coefficient indicated by index "L"), which corresponds to low-level of doping; parameter γ depends on properties of materials of H and could be integer usually in the interval $\gamma \in [1, 3]$ [3]. The equation could be solved numerically, but analytical solution gives us possibility to obtain functional dependence of dopant concentration on parameters. Numerical solution gives us possibility to obtain only numerical values of dopant concentrations.

Let us determine spatiotemporal distribution of dopant concentration by using the same approach as for determination of spatiotemporal distribution of radiation defects concentration. We transform approximation of dopant diffusion coefficient to the following form: $D_L(x,y,z,T) = D_{0L}[1+\varepsilon\, g_L(x,y,z,T)]$, where D_{0L} is the average value of diffusion coefficient for linear diffusion. After accounting the transformation and approximation (3) we obtain

$$\begin{aligned}\frac{\partial C(x,y,z,t)}{\partial t} &= D_{0L}\frac{\partial}{\partial x}\left\{\left[1+\varepsilon\, g_L(x,y,z,T)\right]\left[1+\xi\frac{C^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)}\right]\frac{\partial C(x,y,z,t)}{\partial x}\right\}\\ &+D_{0L}\frac{\partial}{\partial y}\left\{\left[1+\varepsilon\, g_L(x,y,z,T)\right]\left[1+\xi\frac{C^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)}\right]\frac{\partial C(x,y,z,t)}{\partial y}\right\}\\ &+D_{0L}\frac{\partial}{\partial z}\left\{\left[1+\varepsilon\, g_L(x,y,z,T)\right]\left[1+\xi\frac{C^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)}\right]\frac{\partial C(x,y,z,t)}{\partial z}\right\}.\end{aligned} \tag{1a}$$

Further we determine solution of the Eq. (1) as the following power series

$$C(x,t) = \sum_{i=0}^{\infty}\varepsilon^i\sum_{\xi=1}^{\infty}\xi^j C_{ij}(x,y,z,t). \tag{4}$$

Let us substitute the series (4) into Eq. (1a). The substitution gives us possibility to obtain the Eq. (1a) in the following form

$$\begin{aligned}\sum_{i=0}^{\infty}\varepsilon^i\sum_{\xi=1}^{\infty}\xi^j\frac{\partial C_{ij}(x,y,z,t)}{\partial t} &= D_{0L}\frac{\partial}{\partial x}\left(\left[1+\varepsilon g_L(x,y,z,T)\right]\left\{1+\frac{\xi}{P^\gamma(x,y,z,T)}\left[\sum_{i=0}^{\infty}\varepsilon^i\sum_{\xi=1}^{\infty}\xi^j C_{ij}(x,y,z,t)\right]^\gamma\right\}\right.\\ &\left.\times\sum_{i=0}^{\infty}\varepsilon^i\sum_{\xi=1}^{\infty}\xi^j\frac{\partial C_{ij}(x,y,z,t)}{\partial x}\right)+D_{0L}\frac{\partial}{\partial y}\left(\left[1+\varepsilon g_L(x,y,z,T)\right]\left\{1+\frac{\xi}{P^\gamma(x,y,z,T)}\left[\sum_{i=0}^{\infty}\varepsilon^i\sum_{\xi=1}^{\infty}\xi^j C_{ij}(x,y,z,t)\right]^\gamma\right\}\right.\\ &\left.\times\sum_{i=0}^{\infty}\varepsilon^i\sum_{\xi=1}^{\infty}\xi^j\frac{\partial C_{ij}(x,y,z,t)}{\partial y}\right)+D_{0L}\frac{\partial}{\partial z}\left(\left[1+\varepsilon g_L(x,y,z,T)\right]\left\{1+\frac{\xi}{P^\gamma(x,y,z,T)}\left[\sum_{i=0}^{\infty}\varepsilon^i\sum_{\xi=1}^{\infty}\xi^j C_{ij}(x,y,z,t)\right]^\gamma\right\}\right.\\ &\left.\times\sum_{i=0}^{\infty}\varepsilon^i\sum_{\xi=1}^{\infty}\xi^j\frac{\partial C_{ij}(x,y,z,t)}{\partial z}\right).\end{aligned} \tag{1b}$$

Equating coefficients for equal powers of parameters ε and ξ in different parts of Eq. (1b) gives us possibility to obtain equations for zero-order approximation of dopant concentration $C_{00}(x,y,z,t)$ and corrections to it $C_{ij}(x,y,z,t)$. The equations are presented in the Appendix. The analogous procedure we used to obtain boundary and initial conditions for all functions $C_{ij}(x,y,z,t)$, $i \geq 0$, $j \geq 0$. The conditions are also presented in the Appendix. The equations for all functions $C_{ij}(x,y,z,t)$, $i \geq 0$, $j \geq 0$ have been solved by standard Fourier approach [12]. The solutions are presented in the Appendix. It should be noted, that the main idea of considered approach to solve Eq. (1) is usually used for small values of the parameters ε and ξ. However, this modification of the approach gives us possibility any variations of diffusion coefficient in space and time and wide interval of nonlinearity of diffusion process due to native limitation on value of diffusion coefficient.

Analysis of spatiotemporal distributions of dopant concentrations has been done by using the second-order approximation of dopant concentration. Farther the distribution has been amended numerically.

4. DISCUSSION

Let us consider spatial distribution of dopant for different values of difference between diffusion coefficients in layers of H and annealing of time (see Figs. 5–8, respectively). The figures show, that increasing of difference between values of diffusion coefficient gives us possibility to increase sharpness of *p-n*-junction. With increasing of annealing time homogeneity of dopant distribution increases, but sharpness of *p-n*-junction decreases. Existence of interface between layers of H gives us possibility to increase sharpness of *p-n*-junction after annealing with appropriate continuance.

To obtain compromise between increasing of homogeneity of dopant distribution and increasing sharpness of *p-n*-junction let us consider recently introduce criterion [10, 11]. Framework the criterion let us approximate spatial distribution of dopant by step-wise function $\psi(x,y,z)$ (see Fig. 9). Further optimal annealing time we obtain by minimization of the following mean-squared error

$$U = \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \left[C(x,y,z,t) - \psi(x,y,z) \right] d\,z\,d\,y\,d\,x.$$

Dependences of optimal annealing time on several parameters are presented on Fig. 10.

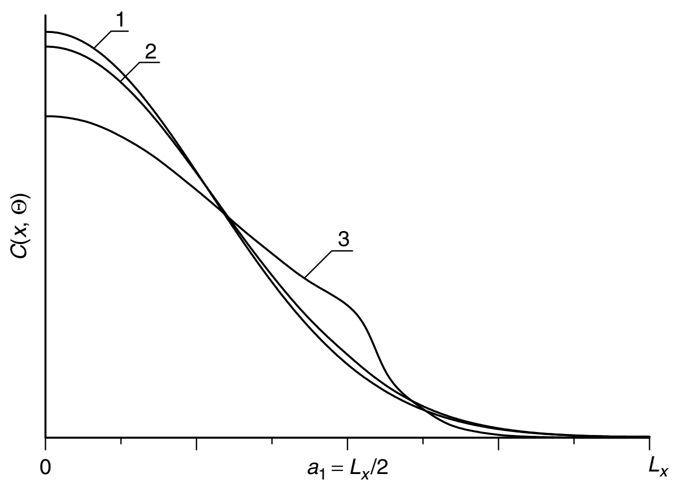

Figure 5. Distribution of dopant in H with two layers (S and one EL) for different values of difference between diffusion coefficient.

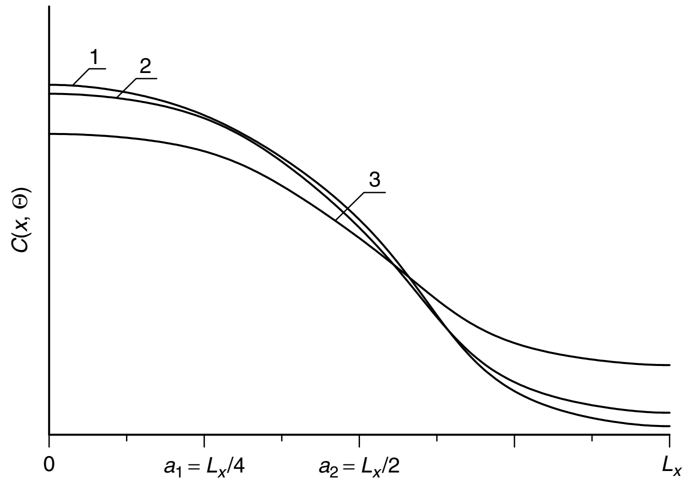

Figure 6. Distribution of dopant in H with three layers (S and two EL) for different values of difference between diffusion coefficient.

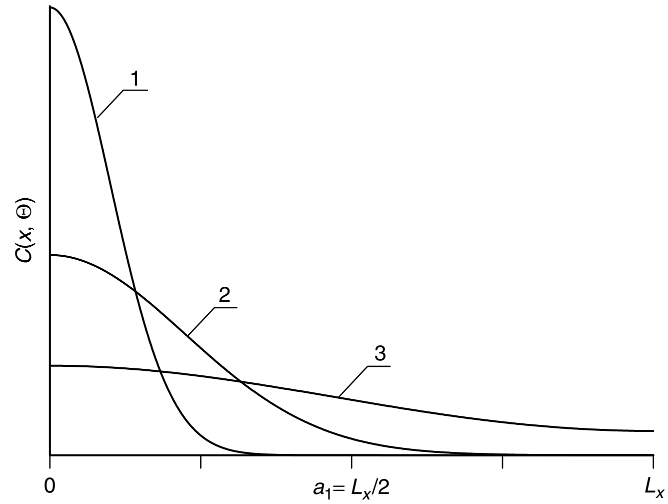

Figure 7. Distribution of dopant in H with two layers (S and one EL) for different values of annealing time.

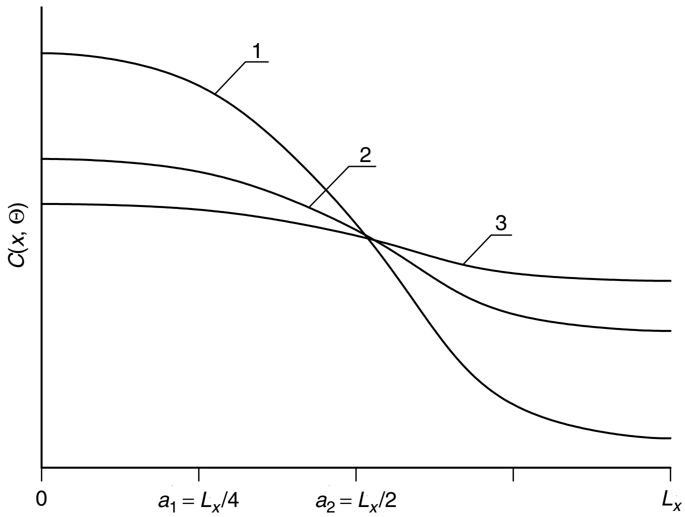

Figure 8. Distribution of dopant in H with three layers (S and two EL) for different values of annealing time.

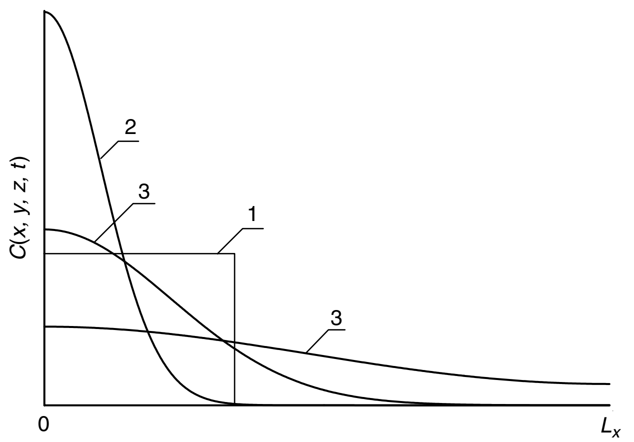

Figure 9. Spatial distributions of dopant in H for diffusion doping. Curve 1 is idealized distribution of dopant. Curves 2–4 are real distributions of dopant for different values of annealing time (increasing of number of curves corresponds to increasing of value of annealing time).

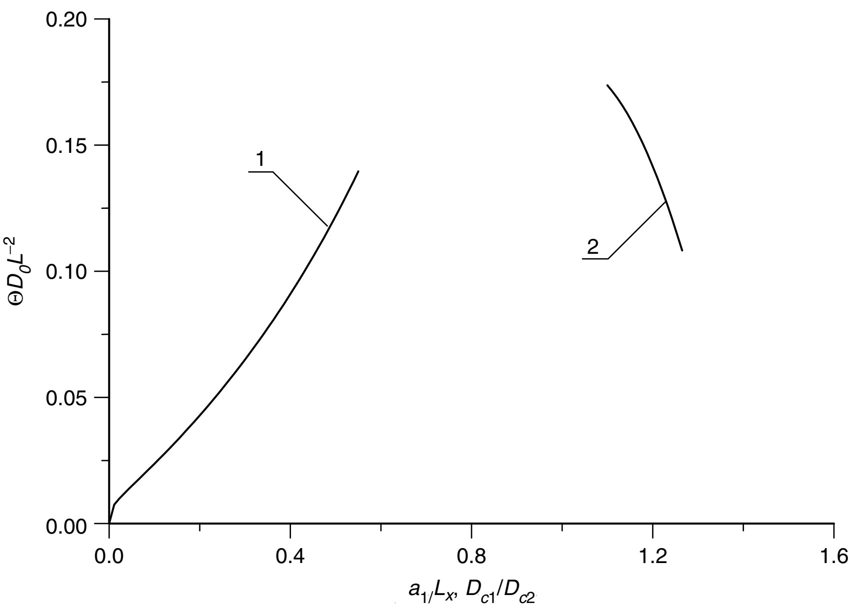

Figure 10. Dependences of dimensionless optimal annealing time of dopant in H on several parameters. Curve 1 is dependence of annealing time on ratio a_1/L_x for $D_1 = D_2 = D_S$. Dependence of annealing time on ratio a_2/L_x is analogous to above dependence. Curve 2 is dependence of annealing time on ratio $D_1 = D_S$ for $a_1/L_x = 1/2$. Dependence of annealing time on ratio $D_2 = D_S$ is analogous to above dependence.

5. CONCLUSION

In this paper it has been elaborated an approach to manufacture more compact field-effect transistor in semiconductor heterostructure. The approach based on using native inhomogeneities of the heterostructure and optimization of technological process.

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APPENDIX

The equations and conditions for the functions $C_{ij}(x,y,z,t)$ ($i \geq 0$, $j \geq 0$) are

$$\frac{\partial C_{00}(x,y,z,t)}{\partial t} = D_{0L}\frac{\partial^2 C_{00}(x,y,z,t)}{\partial x^2} + D_{0L}\frac{\partial^2 C_{00}(x,y,z,t)}{\partial y^2} + D_{0L}\frac{\partial^2 C_{00}(x,y,z,t)}{\partial z^2};$$

$$\frac{\partial C_{i0}(x,y,z,t)}{\partial t} = D_{0L}\frac{\partial^2 C_{i0}(x,y,z,t)}{\partial x^2} + D_{0L}\frac{\partial^2 C_{i0}(x,y,z,t)}{\partial y^2} + D_{0L}\frac{\partial}{\partial x}\left[g_L(x,y,z,T)\frac{\partial C_{i-10}(x,y,z,t)}{\partial x}\right]$$

$$+D_{0L}\frac{\partial}{\partial y}\left[g_L(x,y,z,T)\frac{\partial C_{i-10}(x,y,z,t)}{\partial y}\right] + D_{0L}\frac{\partial}{\partial z}\left[g_L(x,y,z,T)\frac{\partial C_{i-10}(x,y,z,t)}{\partial z}\right]$$

$$+D_{0L}\frac{\partial^2 C_{i0}(x,y,z,t)}{\partial z^2}; i \geq 1;$$

$$\frac{\partial C_{01}(x,y,z,t)}{\partial t} = D_{0L}\frac{\partial^2 C_{01}(x,y,z,t)}{\partial x^2} + D_{0L}\frac{\partial^2 C_{01}(x,y,z,t)}{\partial y^2} + D_{0L}\frac{\partial}{\partial x}\left[\frac{C_{00}^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{00}(x,y,z,t)}{\partial x}\right]$$

$$+D_{0L}\frac{\partial^2 C_{01}(x,y,z,t)}{\partial z^2} + D_{0L}\frac{\partial}{\partial y}\left[\frac{C_{00}^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{00}(x,y,z,t)}{\partial y}\right] + D_{0L}\frac{\partial}{\partial z}\left[\frac{C_{00}^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{00}(x,y,z,t)}{\partial z}\right];$$

$$\frac{\partial C_{02}(x,y,z,t)}{\partial t} = D_{0L}\frac{\partial^2 C_{02}(x,y,z,t)}{\partial x^2} + D_{0L}\frac{\partial}{\partial x}\left[C_{01}(x,y,z,t)\frac{C_{00}^{\gamma-1}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{00}(x,y,z,t)}{\partial x}\right]$$

$$+D_{0L}\frac{\partial^2 C_{02}(x,y,z,t)}{\partial y^2}+D_{0L}\frac{\partial^2 C_{02}(x,y,z,t)}{\partial z^2}+D_{0L}\frac{\partial}{\partial y}\left[C_{01}(x,y,z,t)\frac{C_{00}^{\gamma-1}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{00}(x,y,z,t)}{\partial y}\right]$$

$$+D_{0L}\frac{\partial}{\partial z}\left[C_{01}(x,y,z,t)\frac{C_{00}^{\gamma-1}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{00}(x,y,z,t)}{\partial x}\right]+D_{0L}\frac{\partial}{\partial x}\left[\frac{C_{00}^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{01}(x,y,z,t)}{\partial x}\right]$$

$$+D_{0L}\frac{\partial}{\partial y}\left[\frac{C_{00}^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{01}(x,y,z,t)}{\partial y}\right]+D_{0L}\frac{\partial}{\partial z}\left[\frac{C_{00}^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,t)}\frac{\partial C_{01}(x,y,z,t)}{\partial z}\right];$$

$$\frac{\partial C_{11}(x,y,z,t)}{\partial t}=D_{0L}\frac{\partial^2 C_{11}(x,y,z,t)}{\partial x^2}+D_{0L}\frac{\partial^2 C_{11}(x,y,z,t)}{\partial y^2}+D_{0L}\frac{\partial^2 C_{11}(x,y,z,t)}{\partial z^2}+\frac{\partial}{\partial x}\left[g_L(x,y,z,T)\right.$$

$$\left.\times\frac{\partial C_{01}(x,y,z,t)}{\partial x}\right]D_{0L}+D_{0L}\frac{\partial}{\partial y}\left[g_L(x,y,z,T)\frac{\partial C_{01}(x,y,z,t)}{\partial y}\right]+D_{0L}\frac{\partial}{\partial z}\left[g_L(x,y,z,T)\frac{\partial C_{01}(x,y,z,t)}{\partial z}\right]$$

$$+D_{0L}\frac{\partial}{\partial x}\left[\frac{C_{00}^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{10}(x,y,z,t)}{\partial x}\right]+D_{0L}\frac{\partial}{\partial y}\left[\frac{C_{00}^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{10}(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{\partial C_{10}(x,y,z,t)}{\partial z}\right.$$

$$\left.\times\frac{C_{00}^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\right]D_{0L}+D_{0L}\frac{\partial}{\partial x}\left[C_{10}(x,y,z,t)\frac{C_{00}^{\gamma-1}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{00}(x,y,z,t)}{\partial x}\right]+D_{0L}\frac{\partial}{\partial y}\left[\frac{\partial C_{00}(x,y,z,t)}{\partial y}\right.$$

$$\left.\times C_{10}(x,y,z,t)\frac{C_{00}^{\gamma-1}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\right]+D_{0L}\frac{\partial}{\partial z}\left[C_{10}(x,y,z,t)\frac{C_{00}^{\gamma-1}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\frac{\partial C_{00}(x,y,z,t)}{\partial z}\right];$$

$$\left.\frac{\partial C_{ij}(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial C_{ij}(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial C_{ij}(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial C_{ij}(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,$$

$$\left.\frac{\partial C_{ij}(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial C_{ij}(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,\quad i\ge 0, j\ge 0; C_{00}(x,y,z,0)=f_C(x,y,z), C_{ij}(x,y,z,0)=0, i\ge 1, j\ge 1.$$

The solutions with account boundary and initial conditions could be written as

$$C_{00}(x,y,z,t)=\frac{2\pi}{L_xL_yL_z}\sum_{n=1}^{\infty}F_{nC}C_n(x,y,z)e_{nC}(t)$$

Here $F_{nC}=\int_0^{L_x}c_n(u)\int_0^{L_y}c_n(v)\int_0^{L_z}c_n(w)f_C(u,v,w)\,dw\,dv\,du;\ e_{nC}(t)=\exp\left[-\pi^2n^2D_{0L}\left(L_x^{-2}+L_y^{-2}+L_z^{-2}\right)\right];c_n(s)$

$= \cos(\pi\, n\, s/L_s)$, s is one of spatial coordinates; $s_n(s) = \sin(\pi\, n\, s/L_s)$; $C_n(x,y,z) = c_n(x)\, c_n(y)\, c_n(z)$; u, v and w are variables of integration on spatial coordinates (on x, y or z, respectively);

$$C_{i0}(x,y,z,t) = -\frac{2\pi}{L_x^2 L_y L_z}\sum_{n=1}^{\infty} n F_{nC} C_n(x,y,z)\, e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} s_n(u)\int_0^{L_y} c_n(v)\int_0^{L_z} c_n(w)\frac{\partial C_{i-10}(u,v,w,\tau)}{\partial u}\times$$

$$\times g_L(u,v,w,T)\, d\,w\, d\,v\, d\,u\, d\,\tau - \frac{2\pi}{L_x L_y^2 L_z}\sum_{n=1}^{\infty} n F_{nC} C_n(x,y,z) e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} c_n(u)\int_0^{L_y} s_n(v)\int_0^{L_z} c_n(w)$$

$$\times g_L(u,v,w,T)\frac{\partial C_{i-10}(u,v,w,\tau)}{\partial v}\, d\,w\, d\,v\, d\,u\, d\,\tau - \frac{2\pi}{L_x L_y L_z^2}\sum_{n=1}^{\infty} n F_{nC} C_n(x,y,z) e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} c_n(u)$$

$$\times\int_0^{L_y} c_n(v)\int_0^{L_z} s_n(w)\, g_L(u,v,w,T)\frac{\partial C_{i-10}(u,v,w,\tau)}{\partial w}\, d\,w\, d\,v\, d\,u\, d\,\tau,\; i \ge 1;$$

$$C_{01}(x,y,z,t) = -\frac{2\pi}{L_x^2 L_y L_z}\sum_{n=1}^{\infty} n F_{nC} C_n(x,y,z)\, e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} s_n(u)\int_0^{L_y}\int_0^{L_z}\frac{C_{00}^{\gamma}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}\frac{\partial C_{00}(u,v,w,\tau)}{\partial u}$$

$$\times c_n(v)\, c_n(w)\, d\,w\, d\,v\, d\,u\, d\,\tau - \frac{2\pi}{L_x L_y^2 L_z}\sum_{n=1}^{\infty} n F_{nC} C_n(x,y,z) e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} c_n(u)\int_0^{L_y} s_n(v)\int_0^{L_z}\frac{C_{00}^{\gamma}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}$$

$$\times c_n(w)\frac{\partial C_{00}(u,v,w,\tau)}{\partial v}\, d\,w\, d\,v\, d\,u\, d\,\tau - \frac{2\pi}{L_x L_y L_z^2}\sum_{n=1}^{\infty} n F_{nC}\, C_n(x,y,z) e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} c_n(u)\int_0^{L_y} c_n(v)\int_0^{L_z} s_n(w)$$

$$\times\frac{C_{00}^{\gamma}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}\frac{\partial C_{00}(u,v,w,\tau)}{\partial w}\, d\,w\, d\,v\, d\,u\, d\,\tau;$$

$$C_{02}(x,y,z,t) = -\frac{2\pi}{L_x^2 L_y L_z}\sum_{n=1}^{\infty} C_n(x,y,z) e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} s_n(u)\int_0^{L_y} c_n(v)\int_0^{L_z} c_n(w) C_{01}(u,v,w,\tau)\frac{C_{00}^{\gamma-1}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}$$

$$\times\frac{\partial C_{00}(u,v,w,\tau)}{\partial u}\, d\,w\, d\,v\, d\,u\, d\,\tau\; n F_{nC} - \frac{2\pi}{L_x L_y^2 L_z}\sum_{n=1}^{\infty} n F_{nC} C_n(x,y,z) e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} c_n(u)\int_0^{L_y} s_n(v)\int_0^{L_z} c_n(w)$$

$$\times C_{01}(u,v,w,\tau)\frac{C_{00}^{\gamma-1}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}\frac{\partial C_{00}(u,v,w,\tau)}{\partial v}\, d\,w\, d\,v\, d\,u\, d\,\tau - \frac{2\pi}{L_x L_y L_z^2}\sum_{n=1}^{\infty} F_{nC}\, C_n(x,y,z) e_{nC}(t)\int_0^t e_{nC}(-\tau)$$

$$\times n\int_0^{L_x} c_n(u)\int_0^{L_y} c_n(v)\int_0^{L_z} s_n(w) C_{01}(u,v,w,\tau)\frac{C_{00}^{\gamma-1}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}\frac{\partial C_{00}(u,v,w,\tau)}{\partial w}\, d\,w\, d\,v\, d\,u\, d\,\tau - \frac{2\pi}{L_x^2 L_y L_z}\sum_{n=1}^{\infty} n F_{nC}$$

$$\times C_n(x,y,z) e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} s_n(u)\int_0^{L_y} c_n(v)\int_0^{L_z} c_n(w)\frac{C_{00}^{\gamma}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}\frac{\partial C_{01}(u,v,w,\tau)}{\partial u}\, d\,w\, d\,v\, d\,u\, d\,\tau - \frac{2\pi}{L_x L_y^2 L_z}$$

$$\times\sum_{n=1}^{\infty} n F_{nC} C_n(x,y,z) e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} s_n(u)\int_0^{L_y} c_n(v)\int_0^{L_z} c_n(w)\frac{C_{00}^{\gamma}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}\frac{\partial C_{01}(u,v,w,\tau)}{\partial v}\, d\,w\, d\,v\, d\,u\, d\,\tau$$

$$-\frac{2\pi}{L_x L_y L_z^2}\sum_{n=1}^{\infty} n F_{nC} C_n(x,y,z)\int_0^t e_{nC}(-\tau)\int_0^{L_x} s_n(u)\int_0^{L_y} c_n(v)\int_0^{L_z} c_n(w)\frac{C_{00}^{\gamma}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}\frac{\partial C_{01}(u,v,w,\tau)}{\partial w}\,dw\,dv\,du\,d\tau;$$

$$C_{11}(x,y,z,t)=-\frac{2\pi}{L_x^2 L_y L_z}\sum_{n=1}^{\infty} n F_{nC} C_n(x,y,z) e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} s_n(u)\int_0^{L_y} c_n(v)\int_0^{L_z} g_L(u,v,w,T)\frac{\partial C_{01}(u,v,w,\tau)}{\partial u}$$

$$\times c_n(w)\,dw\,dv\,du\,d\tau-\frac{2\pi}{L_x L_y^2 L_z}\sum_{n=1}^{\infty} F_{nC} C_n(x,y,z)\int_0^t e_{nC}(-\tau)\int_0^{L_x} c_n(u)\int_0^{L_y} s_n(v)\int_0^{L_z} g_L(u,v,w,T)\frac{\partial C_{01}(u,v,w,\tau)}{\partial v}$$

$$\times c_n(w)\,dw\,dv\,du\,d\tau\, n\, e_{nC}(t)-\frac{2\pi}{L_x L_y L_z^2}\sum_{n=1}^{\infty} n F_{nC} C_n(x,y,z) e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} c_n(u)\int_0^{L_y} c_n(v)\int_0^{L_z} g_L(u,v,w,T)$$

$$\times s_n(w)\frac{\partial C_{01}(u,v,w,\tau)}{\partial w}\,dw\,dv\,du\,d\tau-\frac{2\pi}{L_x^2 L_y L_z}\sum_{n=1}^{\infty} n F_{nC} C_n(x,y,z) e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} s_n(u)\int_0^{L_y} c_n(v)\int_0^{L_z} c_n(w)$$

$$\times\frac{C_{00}^{\gamma}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}\frac{\partial C_{10}(u,v,w,\tau)}{\partial u}\,dw\,dv\,du\,d\tau-\frac{2\pi}{L_x L_y^2 L_z}\sum_{n=1}^{\infty} n F_{nC} C_n(x,y,z) e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} c_n(u)\int_0^{L_y} s_n(v)$$

$$\times\int_0^{L_z} c_n(w)\frac{C_{00}^{\gamma}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}\frac{\partial C_{10}(u,v,w,\tau)}{\partial v}\,dw\,dv\,du\,d\tau-\frac{2\pi}{L_x L_y L_z^2}\sum_{n=1}^{\infty} F_{nC} C_n(x,y,z) e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} c_n(u)$$

$$\times n\int_0^{L_y} c_n(v)\int_0^{L_z} sc_n(w)\frac{C_{00}^{\gamma}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}\frac{\partial C_{10}(u,v,w,\tau)}{\partial w}\,dw\,dv\,du\,d\tau-\frac{2\pi}{L_x^2 L_y L_z}\sum_{n=1}^{\infty} n C_n(x,y,z) e_{nC}(t)\int_0^t e_{nC}(-\tau)$$

$$\times F_{nC}\int_0^{L_x} s_n(u)\int_0^{L_y} c_n(v)\int_0^{L_z} c_n(w) C_{10}(u,v,w,\tau)\frac{C_{00}^{\gamma-1}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}\frac{\partial C_{00}(u,v,w,\tau)}{\partial u}\,dw\,dv\,du\,d\tau-\frac{2\pi}{L_x^2 L_y L_z}\sum_{n=1}^{\infty} e_{nC}(t)$$

$$\times n C_n(x,y,z)\int_0^t e_{nC}(-\tau)\int_0^{L_x} s_n(u)\int_0^{L_y} c_n(v)\int_0^{L_z} c_n(w) C_{10}(u,v,w,\tau)\frac{C_{00}^{\gamma-1}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}\frac{\partial C_{00}(u,v,w,\tau)}{\partial u}\,dw\,dv\,du\,d\tau$$

$$\times F_{nC}-\frac{2\pi}{L_x L_y^2 L_z}\sum_{n=1}^{\infty} C_n(x,y,z) e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} s_n(u)\int_0^{L_y} c_n(v)\int_0^{L_z} C_{10}(u,v,w,\tau)\frac{C_{00}^{\gamma-1}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}\frac{\partial C_{00}(u,v,w,\tau)}{\partial v}$$

$$\times c_n(w)\,dw\,dv\,du\,d\tau\, n F_{nC}-\frac{2\pi}{L_x L_y L_z^2}\sum_{n=1}^{\infty} n C_n(x,y,z) e_{nC}(t)\int_0^t e_{nC}(-\tau)\int_0^{L_x} s_n(u)\int_0^{L_y} c_n(v)\int_0^{L_z} c_n(w) C_{10}(u,v,w,\tau)$$

$$\times F_{nC}\frac{C_{00}^{\gamma-1}(u,v,w,\tau)}{P^{\gamma}(u,v,w,T)}\frac{\partial C_{00}(u,v,w,\tau)}{\partial u}\,dw\,dv\,du\,d\tau.$$