Decreasing of Quantity of Radiation Defects in an Implanted-Junction Rectifier in a Semiconductor Heterostructure

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ABSTRACT
It has been recently shown, that manufacturing of an implanted-junction rectifier in a semiconductor heterostructure for optimal relation between annealing time of radiation defects, materials of the heterostructure and thicknesses of layers of the heterostructure and energy of implanted ions leads to increasing of sharpness of $p-n$–junction and at the same time to increasing of homogeneity of dopant distribution in doped area. In this paper we consider an approach of annealing of radiation defects to decrease quantity of radiation defects in comparison with standard of annealing. PACS number(s): 72.20. $-i$; 73.40.Kp; 73.40.Lq; 66.30. $-h$; 85.40.Ry

1. INTRODUCTION

During radiation exposure on semiconductor materials (ion implantation, radiation modification, et al) generation of radiation defects is obtained [1–3]. Due to the radiation damage electrophysical properties of the devises, based on the damaged materials, are changed [4, 5]. Annealing of radiation defects is done to decrease their quantity. In the present time some approaches of annealing are used [4, 6−8]. In this paper an approach of annealing of radiation defects for additional decreasing of their quantity is considered. Framework the approach we consider a semiconductor heterostructure (SH), which consist of a substrate (S) with known type of conductivity (n or p) and epitaxial layer (EL). This SH is presented in Fig. 1. A dopant has been implanted in the EL to produce reverse type of conductivity (p or n). Farther annealing of radiation defects has been done. During the annealing dopant distribution spreads. If the dopant diffusion during the annealing finished near interface between layers of SH, sharpness of the $p-n$–junction increases. At the same time homogeneity of dopant distribution also increases (see Fig. 2 and Refs. [9, 10]). Main aim of the present paper is comparison of obtained in [9, 10] results of analysis of redistribution of dopant and radiation defects with their redistribution in the following situation: in the first step of production of implanted-junction rectifier the dopant has been partially implanted (for example, it has been implanted only one half of dopant). After this implantation radiation defects have been annealed. In the first step another part of dopant have been implanted. After implantation of the second part of dopant radiation defects have been also annealed.

2. METHOD OF SOLUTION

To solve our aims let us determine spatiotemporal distribution of dopant concentration. The distribution we determine by solving the second Fick's law [1, 4, 5]

$$\frac{\partial C(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[D\frac{\partial C(x,t)}{\partial x}\right] \tag{1}$$

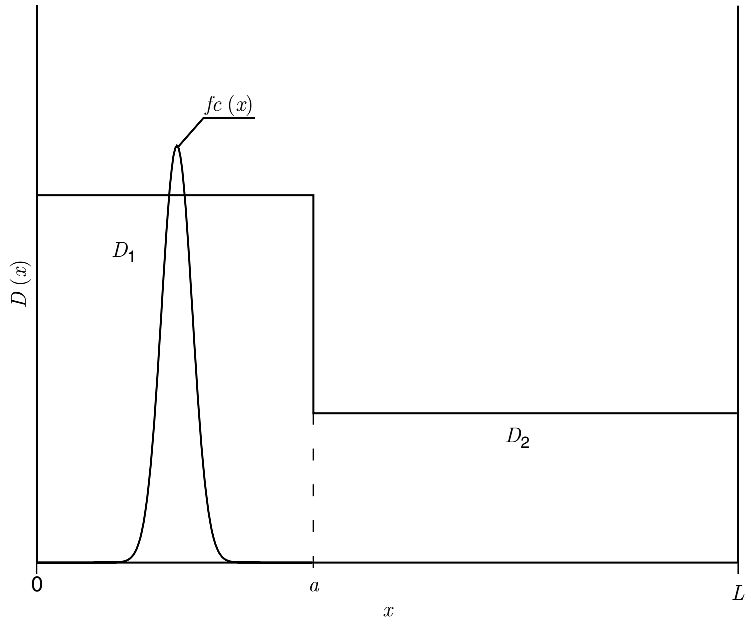

Figure 1. Heterostructure with the epitaxial layer ($x \in [0,\ a]$) and the substrate ($x \in [a,\ L]$). The figure also illustrates initial (before starting of annealing) distribution of radiation defects.

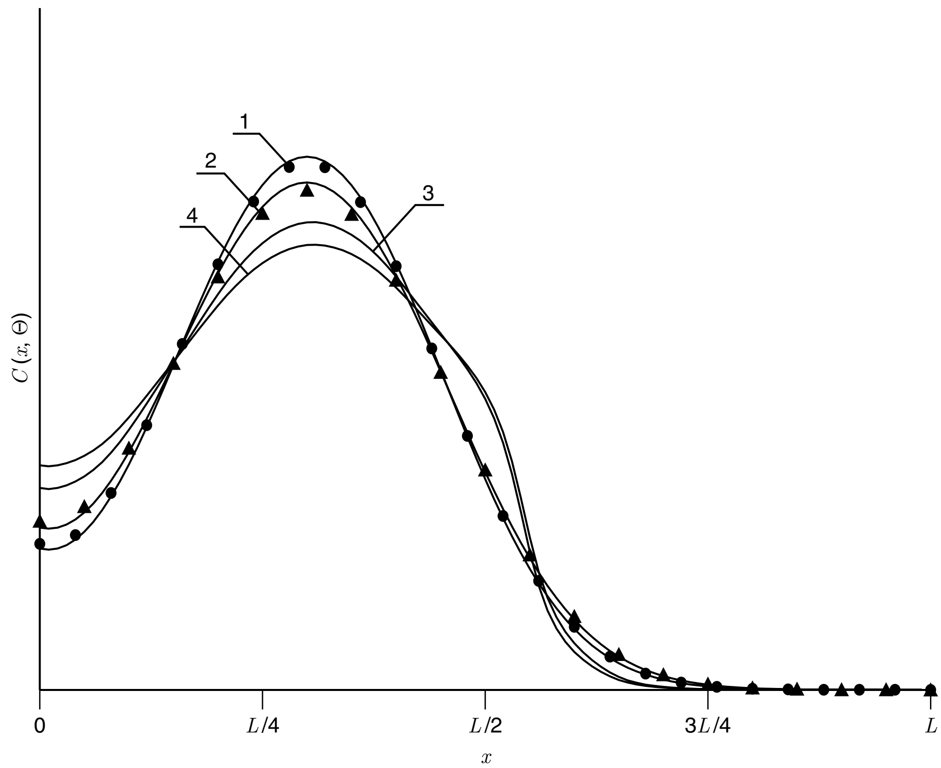

Figure 2. Dopant distributions after annealing of radiation defects with continuance $\Theta = 0.0048L^2/D_0$ (curves 1 and 3) and $\Theta = 0.0057L^2/D_0$ (curves 2 and 4), where D_0 is average value of diffusion coefficient. Circles and triangles are experimental data from [11, 12]. Interface between layers of SH has coordinate $a = L/2$.

with boundary and initial conditions

$$\left.\frac{\partial C(x,t)}{\partial x}\right|_{x=0}=0, \left.\frac{\partial C(x,t)}{\partial x}\right|_{x=L}=0, C(x,0)=f_C(x). \tag{2}$$

Here $C(x, t)$ is spatiotemporal distribution of dopant concentration; D_C is the dopant diffusion coefficient. Value of dopant diffusion coefficient depends on properties of materials of layers in SH, on rate of heating and cooling of SH and on spatiotemporal distribution of dopant and radiation defects concentrations. Concentrational dependence of diffusion coefficient could be approximated by the following functions [13, 14]

$$D_C=D_L(x,T)\left[1+\xi\frac{C^\gamma(x,t)}{P^\gamma(x,T)}\right]\left[1+\zeta_1\frac{V(x,t)}{V^*}+\zeta_2\frac{V^2(x,t)}{(V^*)^2}\right]. \tag{3}$$

Here $D_L(x, T)$ is the diffusion coefficients for low-level of doping; T is the temperature of annealing; parameter γ depends on properties of materials of SH and could be integer usually in the interval $\gamma \in [1, 3]$ [13]; $P(x, T)$ is limit of solubility of dopant in SH; $V(x, t)$ is the spatiotemporal distribution of concentration of vacancies; V^* is equilibrium distribution of concentration of vacancies. Concentrational dependence of diffusion has been discussed in details in [13]. Dependence of dopant diffusion coefficient on concentration of vacancies is generalization of appropriate approximation in [14]. Quadratic term in the approximation corresponds to taking account generation of divacancies (see, for example, [1]). Spatiotemporal distribution of radiation defects has been described by the following system of equations [1, 15, 16]

$$\begin{cases}\dfrac{\partial I(x,t)}{\partial t}=\dfrac{\partial}{\partial x}\left[D_I\dfrac{\partial I(x,t)}{\partial x}\right]-k_{I,V}I(x,t)V(x,t)-k_{I,I}I^2(x,t)\\ \dfrac{\partial V(x,t)}{\partial t}=\dfrac{\partial}{\partial x}\left[D_V\dfrac{\partial V(x,t)}{\partial x}\right]-k_{I,V}I(x,t)V(x,t)-k_{V,V}V^2(x,t)\end{cases} \tag{4}$$

with boundary and initial conditions

$$\begin{aligned}&\left.\frac{\partial I(x,t)}{\partial x}\right|_{x=0}=0,\quad \left.\frac{\partial I(x,t)}{\partial x}\right|_{x=L}=0,\quad \left.\frac{\partial V(x,t)}{\partial x}\right|_{x=0}=0,\quad \left.\frac{\partial V(x,t)}{\partial x}\right|_{x=L}=0,\\ &I(x,0)=f_I(x), V(x,0)=f_V(x).\end{aligned} \tag{5}$$

Here $I(x, t)$ is spatiotemporal distribution of concentration of interstitials; I^* is equilibrium distribution of interstitials; $D_I\ (x, T)$ and $D_V(x, T)$ are diffusion coefficients of interstitials and vacancies, respectively; terms $I^2(x, t)$ and $V^2(x, t)$ correspond to generation of simplest complexes of defects (see [1] and appropriate references in the book); $k_{I,V}(x, T)$, $k_{I,I}(x, T)$ and $k_{V,V}(x, T)$ are parameters of recombination of point radiation defects and generation of their complexes.

Spatiotemporal distributions of concentrations of complexes of radiation defects $\Phi_I(x, t)$ and $\Phi_V(x, t)$ we determined by solution of the following system of equations [1, 15, 16]

$$\begin{cases} \dfrac{\partial \Phi_I(x,t)}{\partial t} = \dfrac{\partial}{\partial x}\left[D_{\Phi_I}\dfrac{\partial \Phi_I(x,t)}{\partial x}\right] + k_{I,I}I^2(x,t) - k_{I,V}I(x,t) \\ \dfrac{\partial \Phi_V(x,t)}{\partial t} = \dfrac{\partial}{\partial x}\left[D_{\Phi_V}\dfrac{\partial \Phi_V(x,t)}{\partial x}\right] + k_{V,V}V^2(x,t) - k_{I,V}V(x,t) \end{cases} \quad (6)$$

with boundary and initial conditions

$$\left.\frac{\partial \Phi_I(x,t)}{\partial x}\right|_{x=0} = 0, \quad \left.\frac{\partial \Phi_I(x,t)}{\partial x}\right|_{x=L} = 0, \quad \left.\frac{\partial \Phi_V(x,t)}{\partial x}\right|_{x=0} = 0, \quad \left.\frac{\partial \Phi_V(x,t)}{\partial x}\right|_{x=L} = 0, \quad (7)$$
$$\Phi_I(x,0) = f_{\Phi_I}(x), \quad \Phi_V(x,0) = f_{\Phi_V}(x).$$

Here $D_{\Phi_I}(x, T)$ and $D_{\Phi_V}(x, T)$ are diffusion coefficients of complexes of point radiation defects; $k_I(x, T)$ are $k_V(x, T)$ are parameters of decay of complexes of point radiation defects.

To determine spatiotemporal distributions of radiation defects in pursuance of Refs. [17–20] we transform approximations of diffusion coefficients of point defects in the following forms: $D_I(x, T) = D_{0I}[1 + \epsilon_I g_I(x, T)]$ and $D_V(x, T) = D_{0V}[1 + \epsilon_V g_V(x, T)]$, where D_{0I} and D_{0V} are average values of the diffusion coefficients. In the same form we introduce approximations of parameters of recombination of point radiation defects and generation of their complexes: $k_{I,V}(x, T) = k_{0I,V}[1 + \epsilon_{I,V}g_{I,V}(x, T)]$, $k_{I,I}(x, T) = k_{0I,I}[1 + \epsilon_{I,I}g_{I,I}(x, T)]$ and $k_{V,V}(x, T) = k_{0V,V}[1 + \epsilon_{V,V}g_{V,V}(x, T)]$. Let us introduce the following dimensionless variables: $\omega_I = \dfrac{V^* K_{0I,V}L^2}{\sqrt{D_{0I}D_{0V}}}, \omega_V = \dfrac{I^* K_{0I,V}L^2}{\sqrt{D_{0I}D_{0V}}}, \Omega_I = \dfrac{V^* K_{0I,I}L^2}{\sqrt{D_{0I}D_{0V}}}, \Omega_V = \dfrac{I^* K_{0V,V}L^2}{\sqrt{D_{0I}D_{0V}}}, \tilde{I}(x,t) = I(x,t)/I^*, \tilde{V}(x,t) = V(x,t)/V^*, \vartheta = \sqrt{D_{0I}D_{0V}}\,t/L^2, \chi = x/L.$ The introduction leads to the following form of Eqs. (4) and (5)

$$\begin{cases} \dfrac{\partial \tilde{I}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0V}}{D_{0I}}}\dfrac{\partial}{\partial \chi}\left[D_I\dfrac{\partial \tilde{I}(\chi,\vartheta)}{\partial \chi}\right] - \omega_I[1+\epsilon_{I,V}g_{I,V}(\chi,T)]\tilde{I}(\chi,\vartheta)\tilde{V}(\chi,\vartheta) \\ \qquad - \Omega_I[1+\epsilon_{I,I}g_{I,I}(\chi,T)]\tilde{I}^2(\chi,\vartheta) \\ \dfrac{\partial \tilde{V}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0I}}{D_{0V}}}\dfrac{\partial}{\partial \chi}\left[D_V\dfrac{\partial \tilde{V}(\chi,\vartheta)}{\partial \chi}\right] - \omega_V[1+\epsilon_{I,V}g_{I,V}(\chi,T)]\tilde{I}(\chi,\vartheta)\tilde{V}(\chi,\vartheta) \\ \qquad - \Omega_V[1+\epsilon_{V,V}g_{V,V}(\chi,T)]\tilde{V}^2(\chi,\vartheta) \end{cases} \quad (8)$$

with boundary and initial conditions

$$\left.\frac{\partial \tilde{I}(\chi,\vartheta)}{\partial \chi}\right|_{\chi=0} = 0, \quad \left.\frac{\partial \tilde{I}(\chi,\vartheta)}{\partial \chi}\right|_{\chi=1} = 0, \quad \left.\frac{\partial \tilde{V}(\chi,\vartheta)}{\partial \chi}\right|_{\chi=0} = 0, \quad \left.\frac{\partial \tilde{V}(\chi,\vartheta)}{\partial \chi}\right|_{\chi=1} = 0, \quad (9)$$
$$\tilde{I}(\chi,0) = \frac{f_I(\chi)}{I^*}, \tilde{V}(\chi,0) = \frac{f_V(\chi)}{V^*}.$$

Let us determine solutions of Eqs. (8) with conditions Eqs. (9), in pursuance of Refs. [17–20], as the following power series

$$\tilde{I}(\chi,\vartheta)=\sum_{i=0}^{\infty}\epsilon_I^i\sum_{j=0}^{\infty}\omega_I^j\sum_{k=0}^{\infty}\Omega_I^k\tilde{I}_{ijk}(\chi,\vartheta),\ \tilde{V}(\chi,\vartheta)=\sum_{i=0}^{\infty}\epsilon_V^i\sum_{j=0}^{\infty}\omega_V^j\sum_{k=0}^{\infty}\Omega_V^k\tilde{V}_{ijk}(\chi,\vartheta). \tag{10}$$

Substitution of the series Eq. (10) into Eqs. (8) and conditions Eqs. (9) gives us possibility to obtain equations for zeroth-order approximations of point defects concentrations $\tilde{I}_{000}(\chi,\vartheta)$, $\tilde{V}_{000}(\chi,\vartheta)$ and corrections for them $\tilde{I}_{ijk}(\chi,\vartheta)$, $\tilde{V}_{ijk}(\chi,\vartheta)$ for $i \geq 1$, $j \geq 1$ and $k \geq 1$. The equations and conditions are presented in the Appendix. Equations of the system (12) could be solved by standard approaches of the mathematical physics (see, for example, Refs. [21]. The solutions with account boundary and initial conditions are presented in the Appendix.

Further we determine spatiotemporal distributions of complexes of radiation defects. To determine the distributions we transform the diffusion coefficients in the following form: $D_{\Phi I}(x,T) = D_{0\Phi I}[1 + \epsilon_{\Phi I}g_{\Phi I}(x,T)]$, $D_{\Phi V}(x,T) = D_{0\Phi V}[1 + \epsilon_{\Phi V}g_{\Phi V}(x,T)]$. In this situation the Eqs. (6) takes the form

$$\begin{cases}\dfrac{\partial\Phi_I(x,t)}{\partial t}=D_{0\Phi I}\dfrac{\partial}{\partial x}\left\{[1+\epsilon_{\Phi I}g_{\Phi I}(x,T)]\dfrac{\partial\Phi_I(x,t)}{\partial x}\right\}+k_{I,I}I^2(x,t)-k_I I(x,t)\\ \dfrac{\partial\Phi_V(x,t)}{\partial t}=D_{0\Phi V}\dfrac{\partial}{\partial x}\left\{[1+\epsilon_{\Phi V}g_{\Phi V}(x,T)]\dfrac{\partial\Phi_V(x,t)}{\partial x}\right\}+k_{V,V}V^2(x,t)-k_V V(x,t).\end{cases}$$

Let us determine solution of the equations as the power series

$$\Phi_\rho(x,t)=\sum_{i=0}^{\infty}\epsilon_{\Phi\rho}\Phi_{\rho i}(x,t). \tag{11}$$

Substitution of the series Eqs. (12) into Eqs. (6) and appropriate boundary and initial conditions gives us possibility to zero-order approximations of concentrations of complexes of radiation defects, corrections and conditions for them in the following form

$$\begin{cases}\dfrac{\partial\Phi_{I0}(x,t)}{\partial t}=D_{0\Phi I}\dfrac{\partial^2\Phi_{I0}(x,t)}{\partial x^2}+k_{I,I}I^2(x,t)-k_{I,V}I(x,t)\\ \dfrac{\partial\Phi_{V0}(x,t)}{\partial t}=D_{0\Phi V}\dfrac{\partial^2\Phi_{V0}(x,t)}{\partial x^2}+k_{V,V}V^2(x,t)-k_{I,V}V(x,t)\end{cases};$$

$$\begin{cases}\dfrac{\partial\Phi_{Ii}(x,t)}{\partial t}=D_{0\Phi I}\dfrac{\partial^2\Phi_{Ii}(x,t)}{\partial x^2}+D_{0\Phi I}\dfrac{\partial}{\partial x}\left[g_{\Phi I}(x,T)\dfrac{\partial\Phi_{Ii-1}(x,t)}{\partial x}\right]\\ \dfrac{\partial\Phi_{Vi}(x,t)}{\partial t}=D_{0\Phi V}\dfrac{\partial^2\Phi_{Vi}(x,t)}{\partial x^2}+D_{0\Phi V}\dfrac{\partial}{\partial x}\left[g_{\Phi V}(x,T)\dfrac{\partial\Phi_{Vi-1}(x,t)}{\partial x}\right]\end{cases} i\geq 1;$$

$$\left.\frac{\partial\Phi_{Ii}(x,t)}{\partial x}\right|_{x=0}=0,\left.\frac{\partial\Phi_{Ii}(x,t)}{\partial x}\right|_{x=L}=0,\left.\frac{\partial\Phi_{Vi}(x,t)}{\partial x}\right|_{x=0}=0,\left.\frac{\partial\Phi_{Vi}(x,t)}{\partial x}\right|_{x=L}=0, i\geq 0;$$

$$\Phi_{I0}(x,0)=f_{\Phi_I},\Phi_{V0}(x,0)=f_{\Phi_V};\Phi_{Ii}(x,0)=0,\Phi_{Vi}(x,0)=0, i\geq 0.$$

solution of the equations could be written as

$$\begin{cases} \Phi_{I0}(x,t) = F_{0\Phi I} + \dfrac{2}{L}\sum_{n=1}^{\infty} F_{n\Phi I}\cos(\pi n x) e_{\Phi In}(t) \\ \quad \times \int_0^t e_{\Phi In}(-\tau) \int_0^L \cos(\pi n v)\left[k_{I,I}(v,T) I^2(v,\tau) - k_I(v,T) I(v,\tau)\right] dv\, d\tau \\ \Phi_{V0}(x,t) = F_{0\Phi V} + \dfrac{2}{L}\sum_{n=1}^{\infty} F_{n\Phi V}\cos(\pi n x) e_{\Phi Vn}(t) \\ \quad \times \int_0^t e_{\Phi Vn}(-\tau) \int_0^L \cos(\pi n v)\left[k_{I,I}(v,T) I^2(v,\tau) - k_I(v,T) I(v,\tau)\right] dv\, d\tau, \end{cases}$$

where $F_{n\Phi\rho} = \int_0^L f_{\Phi\rho}(v)dv, \rho = I, V, e_{\Phi In}(t) = \exp\left(-\pi^2 n^2 \dfrac{D_{0\Phi I} t}{L^2}\right), e_{\Phi Vn}(t) = \exp\left(-\pi^2 n^2 \dfrac{D_{0\Phi V} t}{L^2}\right);$

$$\begin{cases} \Phi_{Ii}(x,t) = -2D_{0\Phi I}\dfrac{\pi}{L} + 2L\sum_{n=1}^{\infty} nF_{n\Phi I}\cos(\pi n x) e_{\Phi In}(t) \\ \quad \times \int_0^t e_{\Phi In}(-\tau) \int_0^L \sin(\pi n x) g_{\Phi I}(v,T) \dfrac{\partial \Phi_{Ii-1}(v,\tau)}{\partial v} dv\, d\tau \\ \Phi_{Vi}(x,t) = -2D_{0\Phi V}\dfrac{\pi}{L} + 2L\sum_{n=1}^{\infty} nF_{n\Phi V}\cos(\pi n x) e_{\Phi Vn}(t) \\ \quad \times \int_0^t e_{\Phi Vn}(-\tau) \int_0^L \sin(\pi n v) g_{\Phi V}(v,T) \dfrac{\partial \Phi_{Vi-1}(v,\tau)}{\partial v} dv\, d\tau. \end{cases} \quad i \geq 1$$

Let us determine spatiotemporal distribution of dopant concentration by using the same approach as for determination of spatiotemporal distribution of radiation defects concentration. We transform approximation of dopant diffusion coefficient to the following form: $D_L(x, T) = D_{0L}[1 + \epsilon_L g_L(x, T)]$. Further we determine solution of the Eq. (1) as the following power series

$$C(x,t) = \sum_{i=0}^{\infty} \epsilon_L^i \sum_{j=0}^{\infty} \xi^j C_{ij}(x,t).$$

Substitution of the series in the Eqs. (1) and (2) gives us possibility to obtain equations for zero-order approximation of dopant concentration $C_{00}(x, t)$, corrections to it $C_{ij}(x, t)$ and boundary and initial conditions to them in the following form

$$\begin{cases}\dfrac{\partial C_{00}(x,t)}{\partial t}=D_{0L}\,\dfrac{\partial^2 C_{00}(x,t)}{\partial^2 x}\\ \dfrac{\partial C_{i0}(x,t)}{\partial t}=D_{0L}\,\dfrac{\partial^2 C_{i0}(x,t)}{\partial^2 x}+D_{0L}\,\dfrac{\partial}{\partial x}\left[g_L(x,T)\dfrac{\partial C_{i-10}(x,t)}{\partial x}\right],\ g\ge 1;\\ \dfrac{\partial C_{01}(x,t)}{\partial t}=D_{0L}\,\dfrac{\partial^2 C_{01}(x,t)}{\partial^2 x}+D_{0L}\,\dfrac{\partial}{\partial x}\left[\dfrac{C_{00}^{\gamma}(x,t)}{P^{\gamma}(x,T)}\dfrac{\partial C_{00}(x,t)}{\partial x}\right];\\ \dfrac{\partial C_{02}(x,t)}{\partial t}=D_{0L}\,\dfrac{\partial^2 C_{02}(x,t)}{\partial^2 x}+D_{0L}\,\dfrac{\partial}{\partial x}\left[\dfrac{C_{00}^{\gamma}(x,t)}{P^{\gamma}(x,T)}\dfrac{\partial C_{01}(x,t)}{\partial x}\right]\\ \qquad +D_{0L}\,\dfrac{\partial}{\partial x}\left[C_{01}(x,t)\dfrac{C_{00}^{\gamma-1}(x,t)}{P^{\gamma}(x,T)}\dfrac{\partial C_{00}(x,t)}{\partial x}\right];\\ \dfrac{\partial C_{11}(x,t)}{\partial t}=D_{0L}\,\dfrac{\partial^2 C_{11}(x,t)}{\partial^2 x}+D_{0L}\,\dfrac{\partial}{\partial x}\left[\dfrac{C_{00}^{\gamma}(x,t)}{P^{\gamma}(x,T)}\dfrac{\partial C_{10}(x,t)}{\partial x}\right]\\ \qquad +D_{0L}\,\dfrac{\partial}{\partial x}\left[C_{10}(x,t)\dfrac{C_{00}^{\gamma-1}(x,t)}{P^{\gamma}(x,T)}\dfrac{\partial C_{00}(x,t)}{\partial x}\right]+D_{0L}\,\dfrac{\partial}{\partial x}\left[g_L(x,T)\dfrac{\partial C_{01}(x,t)}{\partial x}\right];\end{cases}$$

$$\left.\frac{\partial C_{ij}(x,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial C_{ij}(x,t)}{\partial x}\right|_{x=L}=0, i\ge 0, j\ge 0; C_{00}(x,0)=f_C(x); C_{ij}(x,0)=0, i\ge 1, j\ge 1.$$

Solutions of the equations with account boundary and initial conditions could be written as

$$\frac{\partial C_{00}(x,t)}{\partial t}=\frac{F_{0C}}{L}+\frac{2}{L}\sum_{n=1}^{\infty}F_{nC}\cos(\pi n x)e_{nC}(t),$$

where $F_{nC}=\int_0^L c_n(u)f_C(u)du, e_{nC}(t)=\exp\left(-\frac{\pi^2 n^2 D_{0L}t}{L^2}\right);$

$$\frac{\partial C_{i0}(x,t)}{\partial t}=-2\pi\frac{D_{0L}}{L^2}\sum_{n=1}^{\infty}nF_{nC}\cos\left(\frac{\pi n x}{L}\right)e_{nC}(t)$$
$$\times\int_0^t e_{nC}(-\tau)\int_0^L g_L(v,T)\sin\left(\frac{\pi n v}{L}\right)\frac{\partial C_{i-10}(v,\tau)}{\partial v}dv\,d\tau, i\ge 1;$$

$$\frac{\partial C_{01}(x,t)}{\partial t} = -2\pi \frac{D_{0L}}{L^2} \sum_{n=1}^{\infty} nF_{nC} \cos\left(\frac{\pi nx}{L}\right) e_{nC}(t) \times \int_0^t e_{nC}(-\tau) \int_0^L \sin\left(\frac{\pi nv}{L}\right) \frac{C_{00}^{\gamma}(v,\tau)}{P^{\gamma}(v,T)} \frac{\partial C_{00}(v,\tau)}{\partial v} dv\, d\tau;$$

$$\frac{\partial C_{02}(x,t)}{\partial t} = -2\pi \frac{D_{0L}}{L^2} \sum_{n=1}^{\infty} nF_{nC} \cos\left(\frac{\pi nx}{L}\right) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^L \sin\left(\frac{\pi nv}{L}\right) C_{01}(v,\tau) \frac{C_{00}^{\gamma}(v,\tau)}{P^{\gamma}(v,T)} \times \frac{\partial C_{00}(v,\tau)}{\partial v} dv\, d\tau - 2\pi \frac{D_{0L}}{L^2} \sum_{n=1}^{\infty} nF_{nC} \cos\left(\frac{\pi nx}{L}\right) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^L \sin\left(\frac{\pi nv}{L}\right) \frac{C_{00}^{\gamma}(v,\tau)}{P^{\gamma}(v,T)} \times \frac{\partial C_{01}(v,\tau)}{\partial v} dv\, d\tau;$$

$$\frac{\partial C_{11}(x,t)}{\partial t} = -2\pi \frac{D_{0L}}{L^2} \sum_{n=1}^{\infty} nF_{nC} \cos\left(\frac{\pi nx}{L}\right) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^L \sin\left(\frac{\pi nv}{L}\right) C_{01}(v,\tau) \frac{C_{00}^{\gamma}(v,\tau)}{P^{\gamma}(v,T)} \times \frac{\partial C_{01}(v,\tau)}{\partial v} dv\, d\tau - 2\pi \frac{D_{0L}}{L^2} \sum_{n=1}^{\infty} nF_{nC} \cos\left(\frac{\pi nx}{L}\right) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^L \sin\left(\frac{\pi nv}{L}\right) \frac{C_{00}^{\gamma}(v,\tau)}{P^{\gamma}(v,T)} \times \frac{\partial C_{10}(v,\tau)}{\partial v} dv\, d\tau - 2\pi \frac{D_{0L}}{L^2} \sum_{n=1}^{\infty} nF_{nC} \cos\left(\frac{\pi nx}{L}\right) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^L \sin\left(\frac{\pi nv}{L}\right) \frac{C_{00}^{\gamma-1}(v,\tau)}{P^{\gamma}(v,T)} \times C_{10}(v,\tau) \frac{\partial C_{00}(v,\tau)}{\partial v} dv\, d\tau.$$

Analysis of spatiotemporal distributions of dopant and radiation defects concentrations has been done analytically by using the second-order approximation of dopant concentration. Farther the distribution has been amended numerically.

3. DISCUSSION

Let us analyzed redistribution of dopant and radiation defects during annealing. For the analysis we choose so relation between energy of implanted dopant and thickness of EL, that after annealing of radiation defects dopant achieves the interface between layers of SH. In this case we obtain maximal increasing of sharpness of $p - n$–junction in the SH in comparison with $p - n$–junction in homogenous sample. If during the annealing the dopant did not achieved the interface, additional annealing of dopant to shift the $p - n$–junction to the interface is necessary. In this paper we confine oneself to situation, when addition annealing is not necessary. During the analysis we obtain the dopant distributions, which are qualitatively same with the dopant distributions in Fig. 2. Temporal distribution of dopant is standard, i.e. dopant distribution spreads with increasing of annealing time. Fig. 3 illustrates distributions of one type of radiation defects (distributions of both types of radiation defects are similar to each other) for both types of annealing (after implantation of all ions at one time and after implantation of two parts of ions with annealing of both parts). Annealing times for both distributions of defects coincides with each other. Analogous distributions of simplest complexes of radiation defects (for example, divacancies) are presented on Fig. 4. The Figs. 3 and 4 show, that implantation of ions during two technological steps and annealing of defects after every steps leads to decreasing of quantity of point defects and increasing of quantity of complexes of point defects in comparison with implantation of ions during one technological step. But sum of point defects and complexes of point defects decreases.

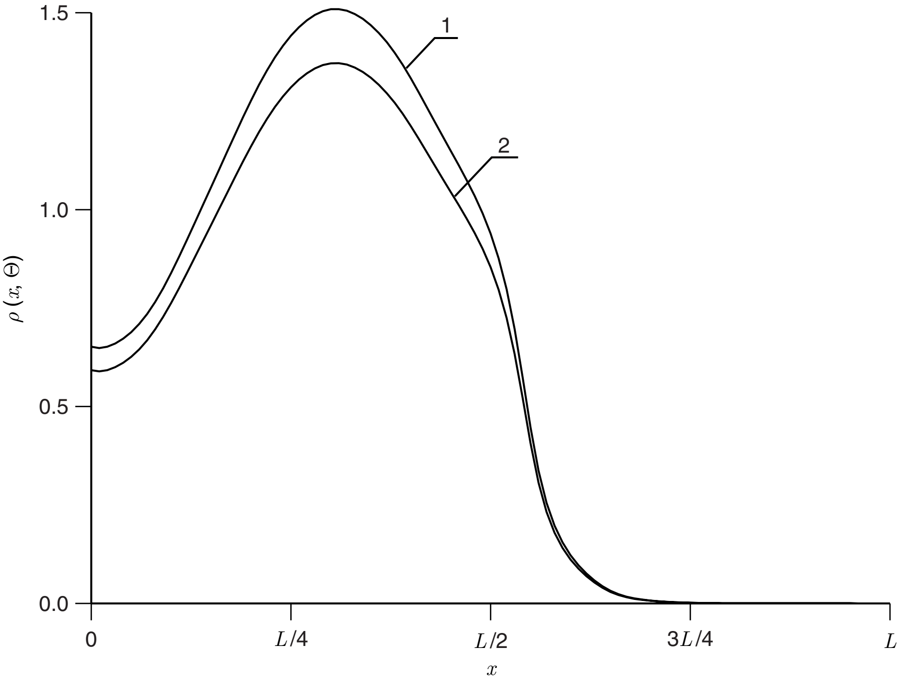

Figure 3. Distributions of radiation defects for fixed value of annealing time. Curve 1 corresponds to implantation of ions during one technological step. Curve 2 corresponds to implantation of ions during two technological steps.

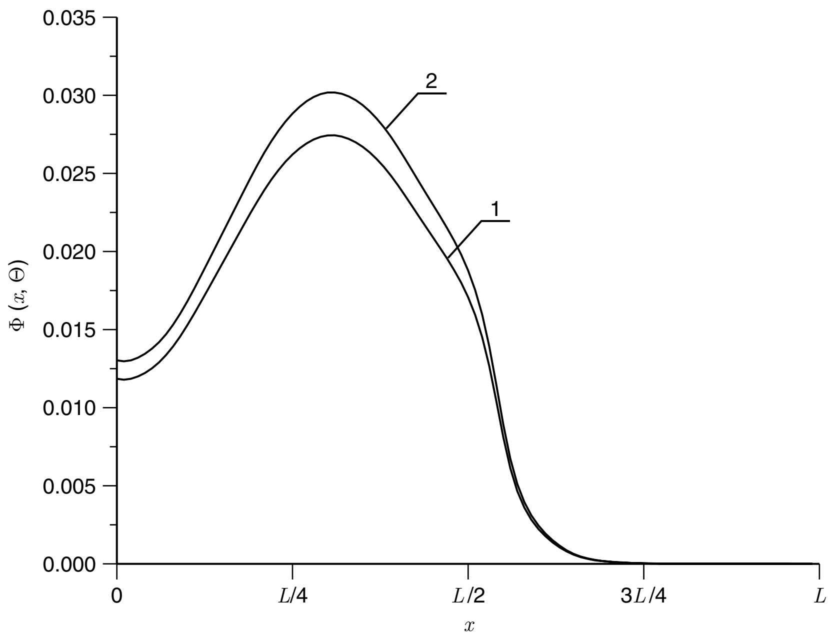

Figure 4. Distributions of complexes radiation defects for fixed value of annealing time. Curve 1 corresponds to implantation of ions during one technological step. Curve 2 corresponds to implantation of ions during two technological steps.

4. CONCLUSION

In this paper we compare two types of annealing of radiation defects, which where produced in semiconductor heterostructure during manufacturing of an implanted-junction rectifier. Type of annealing, which corresponds to decreasing of quantity of radiation defects, has been determined.

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APPENDIX

The equations and conditions for functions $\tilde{I}_{ijk}(\chi, \vartheta)$, $\tilde{V}_{ijk}(\chi, \vartheta)$ for $i \geq 0$, $j \geq 0$ and $k \geq 0$ could be written as

$$\begin{cases} \dfrac{\partial \tilde{I}_{000}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0V}}{D_{0I}}} \dfrac{\partial^2 \tilde{I}_{000}(\chi, \vartheta)}{\partial \chi^2} \\ \dfrac{\partial \tilde{V}_{000}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0I}}{D_{0V}}} \dfrac{\partial^2 \tilde{V}_{000}(\chi, \vartheta)}{\partial \chi^2} \end{cases};$$

$$\begin{cases} \dfrac{\partial \tilde{I}_{i00}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0V}}{D_{0I}}} \dfrac{\partial^2 \tilde{I}_{000}(\chi, \vartheta)}{\partial \chi^2} + \sqrt{\dfrac{D_{0V}}{D_{0I}}} \dfrac{\partial}{\partial \chi}\left[g_I \dfrac{\partial \tilde{I}_{i-100}(\chi, \vartheta)}{\partial \chi}\right] \\ \dfrac{\partial \tilde{V}_{i00}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0I}}{D_{0V}}} \dfrac{\partial^2 \tilde{V}_{000}(\chi, \vartheta)}{\partial \chi^2} + \sqrt{\dfrac{D_{0I}}{D_{0V}}} \dfrac{\partial}{\partial \chi}\left[g_V \dfrac{\partial \tilde{V}_{i-100}(\chi, \vartheta)}{\partial \chi}\right] \end{cases}, \quad i \geq 1;$$

$$\begin{cases} \dfrac{\partial \tilde{I}_{010}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0V}}{D_{0I}}} \dfrac{\partial^2 \tilde{I}_{000}(\chi, \vartheta)}{\partial \chi^2} - [1+\epsilon_{I,V} g_{I,V}(\chi, T)]\tilde{I}_{000}(\chi, \vartheta)\tilde{V}_{000}(\chi, \vartheta) \\ \dfrac{\partial \tilde{V}_{010}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0I}}{D_{0V}}} \dfrac{\partial^2 \tilde{V}_{000}(\chi, \vartheta)}{\partial \chi^2} - [1+\epsilon_{I,V} g_{I,V}(\chi, T)]\tilde{I}_{000}(\chi, \vartheta)\tilde{V}_{000}(\chi, \vartheta) \end{cases};$$

$$\begin{cases} \dfrac{\partial \tilde{I}_{020}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0V}}{D_{0I}}} \dfrac{\partial^2 \tilde{I}_{020}(\chi, \vartheta)}{\partial \chi^2} - [1+\epsilon_{I,V} g_{I,V}(\chi, T)]\tilde{I}_{010}(\chi, \vartheta)\tilde{V}_{000}(\chi, \vartheta) \\ \dfrac{\partial \tilde{V}_{020}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0I}}{D_{0V}}} \dfrac{\partial^2 \tilde{V}_{020}(\chi, \vartheta)}{\partial \chi^2} - [1+\epsilon_{I,V} g_{I,V}(\chi, T)]\tilde{I}_{000}(\chi, \vartheta)\tilde{V}_{010}(\chi, \vartheta) \end{cases};$$

$$\begin{cases} \dfrac{\partial \tilde{I}_{001}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0V}}{D_{0I}}} \dfrac{\partial^2 \tilde{I}_{001}(\chi, \vartheta)}{\partial \chi^2} - [1+\epsilon_{I,I} g_{I,I}(\chi, T)]I^* \tilde{I}^2_{000}(\chi, \vartheta) \\ \dfrac{\partial \tilde{V}_{001}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0I}}{D_{0V}}} \dfrac{\partial^2 \tilde{V}_{001}(\chi, \vartheta)}{\partial \chi^2} - [1+\epsilon_{V,V} g_{V,V}(\chi, T)]V^* \tilde{V}^2_{000}(\chi, \vartheta) \end{cases};$$

$$\begin{cases} \dfrac{\partial \tilde{I}_{002}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0V}}{D_{0I}}} \dfrac{\partial^2 \tilde{I}_{002}(\chi, \vartheta)}{\partial \chi^2} - [1+\epsilon_{I,I} g_{I,I}(\chi, T)]I^* \tilde{I}_{000}(\chi, \vartheta)\tilde{I}_{001}(\chi, \vartheta) \\ \dfrac{\partial \tilde{V}_{002}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\dfrac{D_{0I}}{D_{0V}}} \dfrac{\partial^2 \tilde{V}_{002}(\chi, \vartheta)}{\partial \chi^2} - [1+\epsilon_{V,V} g_{V,V}(\chi, T)]V^* \tilde{V}_{000}(\chi, \vartheta)\tilde{V}_{001}(\chi, \vartheta) \end{cases};$$

$$\begin{cases} \frac{\partial \tilde{I}_{110}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial^2 \tilde{I}_{110}(\chi,\vartheta)}{\partial \chi^2} + \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial}{\partial \chi}\left[g_I(\chi,T) \frac{\partial \tilde{I}_{010}(\chi,\vartheta)}{\partial \chi}\right] \\ \quad -[1+\epsilon_{I,V} g_{I,V}(\chi,T)] V^* \tilde{I}_{100}(\chi,\vartheta) \tilde{V}_{000}(\chi,\vartheta) \\ \frac{\partial \tilde{V}_{110}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial^2 \tilde{V}_{110}(\chi,\vartheta)}{\partial \chi^2} + \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial}{\partial \chi}\left[g_V(\chi,T) \frac{\partial \tilde{V}_{010}(\chi,\vartheta)}{\partial \chi}\right] \\ \quad -[1+\epsilon_{I,V} g_{I,V}(\chi,T)] I^* \tilde{I}_{000}(\chi,\vartheta) \tilde{V}_{100}(\chi,\vartheta) \end{cases};$$

$$\begin{cases} \frac{\partial \tilde{I}_{101}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial^2 \tilde{I}_{101}(\chi,\vartheta)}{\partial \chi^2} + \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial}{\partial \chi}\left[g_I(\chi,T) \frac{\partial \tilde{I}_{001}(\chi,\vartheta)}{\partial \chi}\right] \\ \quad -[1+\epsilon_{I,I} g_{I,I}(\chi,T)] V^* \tilde{I}_{000}(\chi,\vartheta) \tilde{I}_{001}(\chi,\vartheta) \\ \frac{\partial \tilde{V}_{101}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial^2 \tilde{V}_{101}(\chi,\vartheta)}{\partial \chi^2} + \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial}{\partial \chi}\left[g_V(\chi,T) \frac{\partial \tilde{V}_{001}(\chi,\vartheta)}{\partial \chi}\right] \\ \quad -[1+\epsilon_{V,V} g_{V,V}(\chi,T)] I^* \tilde{V}_{000}(\chi,\vartheta) \tilde{V}_{001}(\chi,\vartheta) \end{cases};$$

$$\begin{cases} \frac{\partial \tilde{I}_{011}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial^2 \tilde{I}_{011}(\chi,\vartheta)}{\partial \chi^2} - [1+\epsilon_{I,V} g_{I,V}(\chi,T)] V^* \tilde{I}_{001}(\chi,\vartheta) \tilde{V}_{000}(\chi,\vartheta) \\ \quad -[1+\epsilon_{I,I} g_{I,I}(\chi,T)] V^* \tilde{I}_{000}(\chi,\vartheta) \tilde{I}_{001}(\chi,\vartheta) \\ \frac{\partial \tilde{V}_{101}(\chi,\vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial^2 \tilde{V}_{101}(\chi,\vartheta)}{\partial \chi^2} - [1+\epsilon_{I,V} g_{I,V}(\chi,T)] I^* \tilde{I}_{000}(\chi,\vartheta) \tilde{V}_{001}(\chi,\vartheta) \\ \quad -[1+\epsilon_{V,V} g_{V,V}(\chi,T)] I^* \tilde{V}_{000}(\chi,\vartheta) \tilde{V}_{001}(\chi,\vartheta) \end{cases};$$

$$\left.\frac{\partial \tilde{I}_{ijk}(\chi,\vartheta)}{\partial \chi}\right|_{\chi=0} = 0, \quad \left.\frac{\partial \tilde{I}_{ijk}(\chi,\vartheta)}{\partial \chi}\right|_{\chi=1} = 0; \quad \left.\frac{\partial \tilde{V}_{ijk}(\chi,\vartheta)}{\partial \chi}\right|_{\chi=0} = 0, \quad \left.\frac{\partial \tilde{V}_{ijk}(\chi,\vartheta)}{\partial \chi}\right|_{\chi=1} = 0;$$

$$\tilde{I}_{000}(\chi,0) = \frac{f_I(\chi)}{I^*}, \tilde{V}_{000}(\chi,0) = \frac{f_V(\chi)}{V^*}; \tilde{I}_{ijk}(\chi,0) = 0, \tilde{V}_{ijk}(\chi,0) = 0, i \ge 1, j \ge 1, k \ge 1. \tag{12}$$

The solutions with account boundary and initial conditions could be written as

$$\begin{cases} \tilde{I}_{000}(\chi,\vartheta) = F_{0I} + 2\sum_{i=1}^{\infty} F_{nI} \cos(\pi n \chi) e_{nI}(\vartheta) \\ \tilde{V}_{000}(\chi,\vartheta) = F_{0V} + 2\sum_{i=1}^{\infty} F_{nV} \cos(\pi n \chi) e_{nV}(\vartheta) \end{cases}.$$

Here $F_{n\rho}=\int_0^1 f_\rho(v)dv;\rho=I,V;e_{nI}(\vartheta)=\exp\left(-\pi^2n^2\vartheta\sqrt{\frac{D_{0V}}{D_{0I}}}\right);\ e_{nV}(\vartheta)=\exp\left(-\pi^2n^2\vartheta\sqrt{\frac{D_{0I}}{D_{0V}}}\right);$

$$\begin{cases}\tilde{I}_{i00}(\chi,\vartheta)=-2\pi\sqrt{\frac{D_{0V}}{D_{0I}}}\sum_{i=1}^{\infty}n\cos(\pi n\chi)e_{nI}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\int_0^1\sin(\pi nv)\left[g_I(v,\ T)\frac{\partial\tilde{I}_{i-100}(v,\ \tau)}{\partial\tau}\right]dv\,d\tau\\ \tilde{V}_{i00}(\chi,\vartheta)=-2\pi\sqrt{\frac{D_{0I}}{D_{0V}}}\sum_{i=1}^{\infty}n\cos(\pi n\chi)e_{nV}(\vartheta)\int_0^{\vartheta}e_{nV}(-\tau)\int_0^1\sin(\pi nv)\left[g_V(v,T)\frac{\partial\tilde{V}_{i-100}(v,\ \tau)}{\partial\tau}\right]dv\,d\tau\end{cases} i\geq 1;$$

$$\begin{cases}\tilde{I}_{010}(\chi,\vartheta)=-2\sum_{i=1}^{\infty}\cos(\pi n\chi)e_{nI}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\int_0^1\cos(\pi nv)\left[1+\epsilon_{I,V}g_{I,V}(v,T)\right]\tilde{I}_{000}(v,\ \tau)\tilde{V}_{000}(v,\ \tau)dv\,d\tau\\ \tilde{V}_{010}(\chi,\vartheta)=-2\sum_{i=1}^{\infty}\cos(\pi n\chi)e_{nV}(\vartheta)\int_0^{\vartheta}e_{nV}(-\tau)\int_0^1\cos(\pi nv)\left[1+\epsilon_{I,V}g_{I,V}(v,T)\right]\tilde{I}_{000}(v,\ \tau)\tilde{V}_{000}(v,\ \tau)dv\,d\tau\end{cases};$$

$$\begin{cases}\tilde{I}_{020}(\chi,\vartheta)=-2\sum_{i=1}^{\infty}\cos(\pi n\chi)e_{nI}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\int_0^1\cos(\pi nv)\left[1+\epsilon_{I,V}g_{I,V}(v,T)\right]\tilde{I}_{010}(v,\tau)\tilde{V}_{000}(v,\tau)dv\,d\tau\\ \tilde{V}_{020}(\chi,\vartheta)=-2\sum_{i=1}^{\infty}\cos(\pi n\chi)e_{nV}(\vartheta)\int_0^{\vartheta}e_{nV}(-\tau)\int_0^1\cos(\pi nv)\left[1+\epsilon_{I,V}g_{I,V}(v,T)\right]\tilde{I}_{000}(v,\tau)\tilde{V}_{010}(v,\tau)dv\,d\tau\end{cases};$$

$$\begin{cases}\tilde{I}_{001}(\chi,\vartheta)=-2\sum_{i=1}^{\infty}\cos(\pi n\chi)e_{nI}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\int_0^1\cos(\pi nv)\left[1+\epsilon_{I,I}g_{I,I}(v,T)\right]I^*\tilde{I}^2_{000}(v,\tau)dv\,d\tau\\ \tilde{V}_{001}(\chi,\vartheta)=-2\sum_{i=1}^{\infty}\cos(\pi n\chi)e_{nV}(\vartheta)\int_0^{\vartheta}e_{nV}(-\tau)\int_0^1\cos(\pi nv)\left[1+\epsilon_{V,V}g_{V,V}(v,T)\right]V^*\tilde{V}^2_{000}(v,\ \tau)dv\,d\tau\end{cases};$$

$$\begin{cases}\tilde{I}_{002}(\chi,\vartheta)=-2\sum_{i=1}^{\infty}\cos(\pi n\chi)e_{nI}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\int_0^1\cos(\pi nv)\left[1+\epsilon_{I,I}g_{I,I}(v,T)\right]I^*\tilde{I}_{001}(v,\ \tau)\tilde{I}_{000}(v,\ \tau)dv\,d\tau\\ \tilde{V}_{002}(\chi,\vartheta)=-2\sum_{i=1}^{\infty}\cos(\pi n\chi)e_{nV}(\vartheta)\int_0^{\vartheta}e_{nV}(-\tau)\int_0^1\cos(\pi nv)\left[1+\epsilon_{V,V}g_{V,V}(v,T)\right]V^*\tilde{V}_{001}(v,\ \tau)\tilde{V}_{000}(v,\ \tau)dv\,d\tau\end{cases};$$

$$\begin{cases}\tilde{I}_{110}(\chi,\vartheta)=-2\pi\sqrt{\frac{D_{0V}}{D_{0I}}}\sum_{i=1}^{\infty}n\cos(\pi n\chi)e_{nI}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\int_0^1\sin(\pi nv)\left[g_I(v,T)\frac{\partial\tilde{I}_{000}(v,\ \tau)}{\partial\tau}\right]dv\,d\tau\\ \qquad-2\sum_{i=1}^{\infty}\cos(\pi n\chi)e_{nI}(\vartheta)\int_0^{\vartheta}e_{nI}(-\tau)\int_0^1\cos(\pi nv)\left[1+\epsilon_{I,V}g_{I,V}(v,T)\right]V^*\tilde{I}_{100}(v,\ \tau)\tilde{V}_{000}(v,\ \tau)dv\,d\tau\\ \tilde{V}_{110}(\chi,\vartheta)=-2\pi\sqrt{\frac{D_{0I}}{D_{0V}}}\sum_{i=1}^{\infty}n\cos(\pi n\chi)e_{nV}(\vartheta)\int_0^{\vartheta}e_{nV}(-\tau)\int_0^1\sin(\pi nv)\left[g_V(v,T)\frac{\partial\tilde{V}_{000}(v,\ \tau)}{\partial\tau}\right]dv\,d\tau\\ \qquad-2\sum_{i=1}^{\infty}\cos(\pi n\chi)e_{nV}(\vartheta)\int_0^{\vartheta}e_{nV}(-\tau)\int_0^1\cos(\pi nv)\left[1+\epsilon_{I,V}g_{I,V}(v,T)\right]I^*\tilde{I}_{000}(v,\tau)\tilde{V}_{100}(v,\tau)dv\,d\tau\end{cases};$$

$$\begin{cases} \tilde{I}_{101}(\chi,\vartheta) = -2\pi\sqrt{\dfrac{D_{0V}}{D_{0I}}}\sum_{i=1}^{\infty} n\cos(\pi n\chi)e_{nI}(\vartheta)\int_0^{\vartheta} e_{nI}(-\tau)\int_0^1 \sin(\pi n v)\left[g_I(v,T)\dfrac{\partial \tilde{I}_{001}(v,\tau)}{\partial \tau}\right]dv\,d\tau \\ \qquad -2\sum_{i=1}^{\infty}\cos(\pi n\chi)e_{nI}(\vartheta)\int_0^{\vartheta} e_{nI}(-\tau)\int_0^1 \cos(\pi n v)\left[1+\epsilon_{I,V}g_{I,V}(v,T)\right]V^*\tilde{I}_{100}(v,\tau)\tilde{V}_{000}(v,\tau)dv\,d\tau \\ \tilde{V}_{101}(\chi,\vartheta) = -2\pi\sqrt{\dfrac{D_{0I}}{D_{0V}}}\sum_{i=1}^{\infty} n\cos(\pi n\chi)e_{nV}(\vartheta)\int_0^{\vartheta} e_{nV}(-\tau)\int_0^1 \sin(\pi n v)\left[g_V(v,T)\dfrac{\partial \tilde{V}_{001}(v,\tau)}{\partial \tau}\right]dv\,d\tau \\ \qquad -2\sum_{i=1}^{\infty}\cos(\pi n\chi)e_{nV}(\vartheta)\int_0^{\vartheta} e_{nV}(-\tau)\int_0^1 \cos(\pi n v)\left[1+\epsilon_{I,V}g_{I,V}(v,T)\right]I^*\tilde{I}_{000}(v,\tau)\tilde{V}_{100}(v,\tau)dv\,d\tau \end{cases};$$

$$\begin{cases} \tilde{I}_{011}(\chi,\vartheta) = -2\sum_{i=1}^{\infty}\cos(\pi n\chi)e_{nI}(\vartheta)\int_0^{\vartheta} e_{nI}(-\tau)\int_0^1 \cos(\pi n v)\left[1+\epsilon_{I,V}g_{I,V}(v,T)\right]V^*\tilde{I}_{001}(v,\tau)\tilde{V}_{000}(v,\tau)dv\,d\tau \\ \qquad -2\sum_{i=1}^{\infty}\cos(\pi n\chi)e_{nI}(\vartheta)\int_0^{\vartheta} e_{nI}(-\tau)\int_0^1 \cos(\pi n v)\left[1+\epsilon_{I,I}g_{I,I}(v,T)\right]V^*\tilde{I}_{000}(v,\tau)\tilde{I}_{010}(v,\tau)dv\,d\tau \\ \tilde{V}_{011}(\chi,\vartheta) = -2\sum_{i=1}^{\infty}\cos(\pi n\chi)e_{nV}(\vartheta)\int_0^{\vartheta} e_{nV}(-\tau)\int_0^1 \cos(\pi n v)\left[1+\epsilon_{I,V}g_{I,V}(v,T)\right]V^*\tilde{I}_{001}(v,\tau)\tilde{V}_{000}(v,\tau)dv\,d\tau \\ \qquad -2\sum_{i=1}^{\infty}\cos(\pi n\chi)e_{nV}(\vartheta)\int_0^{\vartheta} e_{nV}(-\tau)\int_0^1 \cos(\pi n v)\left[1+\epsilon_{V,V}g_{V,V}(v,T)\right]V^*\tilde{V}_{000}(v,\tau)\tilde{V}_{010}(v,\tau)dv\,d\tau. \end{cases}$$