Nonlinear Aerodynamic Model Structure Determination using Statistical Measures

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Abstract:
The stepwise regression is a well-established technique used to determine aerodynamic model structure from flight data. This paper discusses the application of stepwise regression to a nonlinear estimation method proposed previously by the authors, for model structure determination. The approach can also be used for updating aerodynamic database models in table look-up form. The current results obtained for model structure determination for a reference problem are compared against the results obtained using the well known least squares multivariate linear estimation. Simulated data of a transport aircraft is used to prove the concept. Finally, an application of the technique for the aero database update of a high performance fighter aircraft is discussed.

Key words: Nonlinear parameter estimation, stepwise regression

1. INTRODUCTION

Identification and determination of model structure/parameters from measured experimental data necessitates use of statistical experimental design fundamentals, regression modeling techniques, and optimization methods. All these three elements are usually combined into a single technique called Response Surface Methodology (RSM). In this paper, we are concerned about the regression modeling techniques for finding adequate nonlinear aerodynamic model structure with standard test for significance like that of *F*-test in RSM. The Least squares estimation, also called regression analysis is well known for its computational simplicity. The regression techniques can be applied to nonlinear models as well since the underlying principle is based upon minimizing the sum of squares of the error between the measurements and model response [1]. The model structure determination and its statistical validation, play a vital role in aerodynamic database update process [2-3]. The important factor in the estimation process is the information content present in the data with respect to the postulated model. Statistical measures are useful in inferring the content of information and hence help to arrive at an adequate model structure. Given a set of data there is always a trade-off between the model structure complexity and accuracy of estimation. Statistical measures help in deciding whether the postulated model structure is adequate for the desired accuracy or the data warrants a change in the model structure.

A brief description of a technique for determining the airplane model structure from flight data using modified stepwise regression [4-5] is given below: The general form of aerodynamic model equations can be written as:

$$y(t) = \theta_0 + \theta_1 x_1(t) + ... + \theta_{g-1} x_{g-1}(t) + \varepsilon \qquad (1)$$

where $y(t)$, represents the resultant coefficient of aerodynamic force or moment (the dependent variable), parameterized by unknownsto θ_0 to $\theta_{g\text{-}1}$ and $x_1(t)$ to $x_{g\text{-}1}$ are the airplane response and input variables and their combinations (the independent variables). Assuming that a sequence of N observations of x and of y has been made, then an adequate model for the aerodynamic coefficients can be determined by applying the stepwise regression [4-5]. If the dependent variable y is linearly related to independent variables, then such a model is called as a linear model, and is valid for smaller excursions around an operating point. Models representing the nonlinear response relationship are valid over a wider range. There are two approaches discussed in the literature to handle such models. They are:

1) Model being linear in parameter but nonlinear in the independent variables [4];
For e.g.,

$$y(t) = \theta_0 + \theta_1 x_1(t) + \theta_2 {x_2}^2(t) + \theta_3 x_1(t) x_2(t) + \ldots\ldots \tag{2}$$

2) Model being nonlinear in the parameter itself [6];
For e.g.,

$$y(t) = f(x(t), \theta) \tag{3}$$

In this paper, we address latter type of models and determine the adequate structure using stepwise regression. The function *f* can be estimated by minimizing the cost function iteratively [1]. The problem of determining the function *f* has been approached in literature using spline functions. A spline function can be fitted to (3) and the coefficients of spline function can be estimated using least squares/ maximum likelihood estimation [2,4,5]. Instead of this approach, we propose to estimate the nonlinear function *f* directly. The algorithmic steps of the proposed estimation technique can be found from [6]. The key to the direct estimation of the nonlinear function *f*, is to represent it in the form of table lookup with linear interpolation. The paper shows that stepwise regression applied to the proposed method for model structure determination requires less computational effort when compared to model structure determination for models represented by (2). The concept is proven using the six-DOF simulated data of a light transport aircraft. Subsequently, it is applied for updating the aerodynamic database of a high performance fighter aircraft.

2. STEPWISE REGRESSION AND STATISTICAL MEASURES

In regression analysis, whenever a relationship between two quantities, y_i and x_i is sought, there is a need for a measure of goodness-of-fit. The F-statistic, also known as the F-ratio is a measure of the strength of the regression. A strong relationship between y_i and x_i gives a high F-ratio [1].

The procedure for stepwise regression as applied to models of form represented by (2) starts with an assumption that there are no variables in the postulated regression equation other than the bias term θ_0. Subsequently an optimal subset of variables is found by inserting independent variables into the model one at a time. The first independent variable entering into the regression equation is the one that has largest correlation with the dependent variable. This variable should also produce the largest value of the F-statistic for testing the significance of regression. The variable is entered if the partial F-statistic exceeds a preselected critical F-value.

$$F_p = \frac{\hat{\theta}_1^2}{s^2(\hat{\theta}_1)} > F_{critical} \tag{4}$$

where $\hat{\theta}_1$ is the estimated parameter associated with x_1 and $s^2(\hat{\theta}_1)$ is the variance estimate of θ_1.

The second variable chosen for the entry should be such that it has the largest correlation with y after accounting for the effect of first variable in y. These correlations are denoted by the partial correlation terms. In general, at each step, the independent variable having highest partial correlation with the dependent variable finds an entry into the regressor provided the partial F-statistic exceeds the prespecified value. Since the optimal choice of variables in the regression is not known apriori, a variable added at an earlier step may be redundant, because the relationship between it and the remaining variables as present in the equation can reduce its F-statistic to less than $F_{critical}$. If this happens, this variable is deleted from the regressor. This procedure terminates after all significant terms have been included in the model. The flow chart for the procedure to determine adequate model structure is given in Figure 1.

The steps in stepwise regression are summarized as follows [1]:

STEP 1: Compute the correlation coefficients between y (dependent variable) and each independent variable x_i, where $i = 1,2,\ldots,p$.

$$r_{yx_i} = \frac{\sum_{k=1}^{N} (y(k) - \bar{y})(x_i(k) - \bar{x}_i)}{\sqrt{\sum_{k=1}^{N} (y(k) - \bar{y})^2 \sum_{k=1}^{N} (x_i(k) - \bar{x}_i)^2}} \tag{5}$$

where $\bar{y} = \frac{1}{N}\sum_{k=1}^{N} y(k)$ and $\bar{x}_i = \frac{1}{N}\sum_{k=1}^{N} x_i(k)$ and k represents discrete points in time.

Check whether the correlation coefficients are significant or not (i.e., greater than a prespecified threshold). If not, none of the potential variables can enter the model, and the mean value yields the best fit. Otherwise, select the independent variable, say the j^{th} variable with the highest correlation coefficient as the first entry and postulate the probable regression model as

$$y = \theta_0 + \theta_j x_j + \varepsilon \tag{6}$$

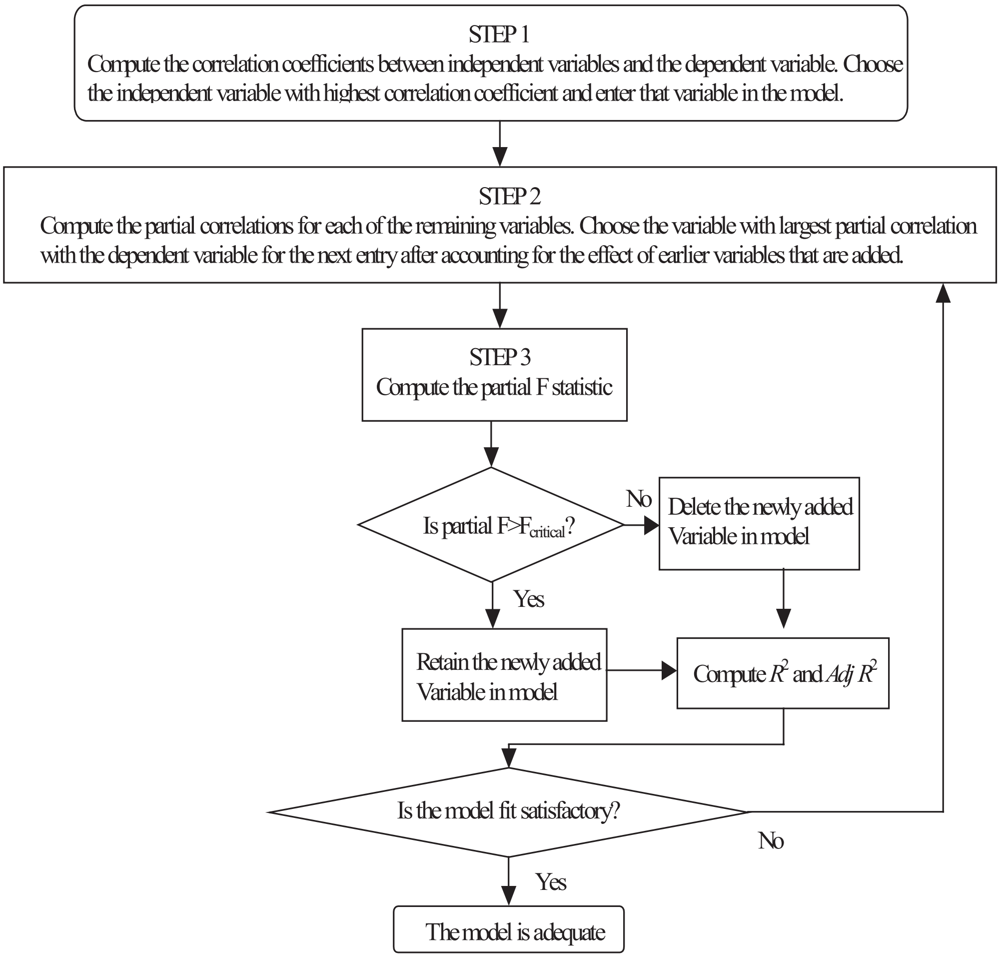

Figure 1. Flow chart depicting the steps in adequate model structure determination

STEP 2: Compute the partial correlations, $r_{yx_i*x_j}$, $i \neq j$, for each of the remaining independent variables ($i = 1,2,\ldots j\text{-}1, j\text{+}1,\ldots,p$.). The procedure to compute the partial correlations with one or more variables held fixed, is as follows:
a) Fit a model dropping the j^{th} independent variable

$$y = \theta^y{}_0 + \theta^y{}_1 x_1 + \ldots.. + \theta^y{}_{j-1} x_{j-1} + \theta^y{}_{j+1} x_{j+1} + \theta^y{}_{nq} x_{nq} + \varepsilon \tag{7}$$

b) Next fit the same model to the j^{th} independent variable which was dropped in (7)

$$x_j = \theta + \theta_1^x x_1 + \ldots.. + \theta_{j-1}^x x_{j-1} + \theta_{j+1}^x x_{j+1} + \theta_{nq}^x x_{nq} + \varepsilon \tag{8}$$

c) Find out the fit errors from (7) and (8). Denote them as e_y and e_x.
d) Calculate the partial correlation using the fit errors as follows:

$$r_{yx_j*(x_1,\ldots,x_{j-1},x_{j+1},\ldots,x_p)} = \frac{\sum_{k=1}^{N}[e_y(k)-\overline{e}_y][e_x(k)-\overline{e}_x]}{\sqrt{\sum_{k=1}^{N}[e_y(k)-\overline{e}_y]^2 \sum_{k=1}^{N}[e_x(k)-\overline{e}_x]^2}} \tag{9}$$

where $\overline{e}_y$ and $\overline{e}_x$ are the respective mean values.

Choose an independent variable with the largest partial correlation, say the k^{th} independent variable. Fit the new model with the new independent variable

$$y = \theta_0 + \theta_j x_j + \theta_k x_k + \varepsilon$$

STEP 3: Compute the partial F values F_j *and* F_k corresponding to the j^{th} and k^{th} variable. Find the minimum F among these two and compare it with $F_{critical}$.
F-statistic from partial correlation

$$F = \frac{r_{yx_i*x_j}}{(1 - r_{yx_i*x_j})} \frac{N-p}{p-1} \tag{10}$$

If $\min(F_j,F_k) < F_{critical}$, it means that the contribution due to the corresponding variable is not significant; accordingly delete the j^{th} or the k^{th} independent variable whichever is smaller and return back to STEP 2.

If $\min(F_j,F_k) > F_{critical}$, it means k^{th} variable has a significant contribution to dependent variable and hence can be retained in the model.

Return back to STEP 2 and examine the partial correlations of y on the remaining variables, retaining j and k in the model.

The procedure automatically stops when the variable once entered in the model cannot be removed and when addition of any new variables does not lead to further improvement in the model.

Some other useful criteria are the coefficient of determination (R^2) and the adjusted coefficient of determination are used for evaluating the model fit.

$$R^2 = \frac{\sum_{k=1}^{N} [\hat{y}(k) - \overline{y}]^2}{\sum_{k=1}^{N} [y(k) - \overline{y}]^2} \tag{11}$$

The value of R^2 lies between 0 and 1. The larger the value of R^2, the better the fit is. However, it tends to be overestimated when the number of samples N is not large when compared with p.

The adjusted coefficient of determination, $AdjR^2$, is a modified version that corrects (11) for the number of parameters being estimated and does not necessarily increase with extra variables.

$$AdjR^2 = 1 - (1 - R^2)\frac{N-1}{N-p-1} \tag{12}$$

3. ADEQUATE MODEL STRUCTURE

Under this section two different model structures are discussed: one is linear in parameter and the other is nonlinear in parameter.

3.1 Models linear in parameters and nonlinear in independent variables

This approach is dealt in [4]. We demonstrate the same using the six-DOF simulated data of light transport aircraft. Let us consider the modeling of pitching moment coefficient. The pitching moment coefficient is the dependent variable here and it depends on independent variables like angle of attack, pitch rate and elevator. The pitching moment coefficient exhibits nonlinearity with respect to angle of attack. Fitting a suitable model structure to this kind of nonlinear relation is an art and a challenge, and this is discussed in this paper.

In general for an aero database update process, the aerodynamic coefficients are computed from flight data using inverse six-DOF equations [7]. Subsequently, the adequate nonlinear model structure is determined using the steps given in Section 2. Table 1 gives the complete analysis of stepwise regression to arrive at adequate model structure. The $F_{critical}$ is assumed to be 50. This value is arrived based on the modeling of flight data off-line on a trial and error basis.

It is noted from Table 1 that α^4 gets eliminated from the model structure at step 8. Figure 2 shows the model fit obtained for the pitching moment coefficient for alternate steps compared with the true value.

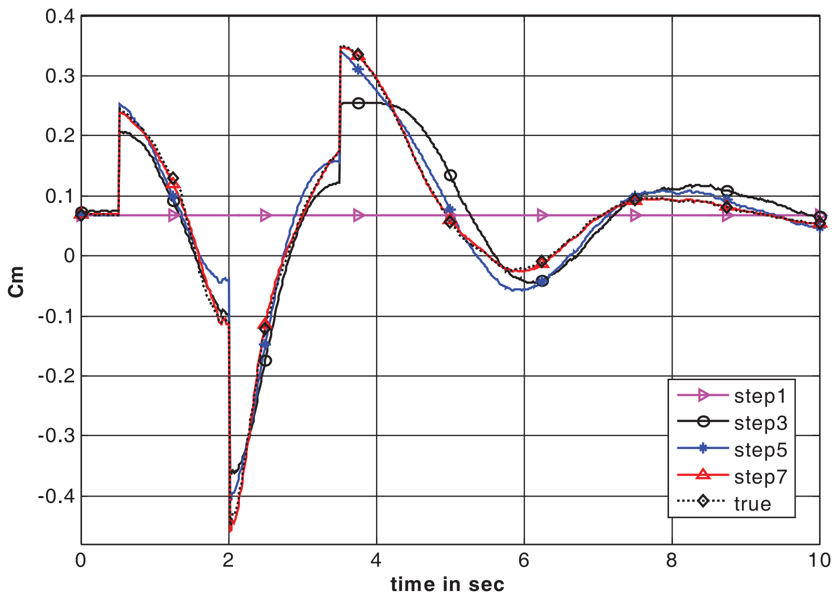

Figure 2. Estimated model compared with the true model (Linear estimation)

Obviously the fit between model arrived at seventh step and the truth is satisfactory as the model is adequate. Figure 3 is a plot showing the convergence of adequate model fit obtained after every step.

It is seen that variance of seventh step is closer to zero, R^2 and $AdjR^2$ values are closer to 1 indicating the model adequacy. It is noted in this case that the values R^2 and $AdjR^2$ are tracking each other because N is larger than p. However, this may not be the case when the number of samples N is smaller when compared with p.

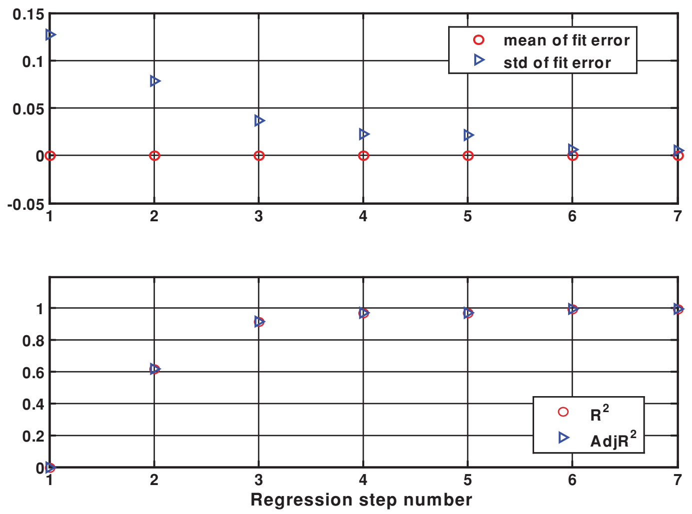

Figure 3. Convergence of adequate model

3.2 Models nonlinear in parameters and nonlinear in independent variables

In the previous Subsection, adequate nonlinear model structure determination is carried out where the model is linear in parameters and nonlinear in independent variables. In this section, adequate model structure determination of models being nonlinear in parameters is addressed. These models can be constructed using splines [2,5]. In this paper we do not use spline / polynomial functions for modeling. The nonlinear function $C_m(\alpha)$ is estimated as a table look-up form directly [6]. The technique makes use of linear interpolation and suitable break points are chosen to capture the nonlinearity. The reasons for postulating this type of a model is because the wind tunnel data used to model the chosen aircraft uses table look ups that has a similar structure, and hence, a one to one direct relation between the estimated and the truth model is established. The proposed estimation technique does not involve any iterative optimization process [4]. It consists of only linear interpolation and least squares estimation to capture the nonlinearity. The criteria for termination was to achieve the same level of $Adj\ R^2$ statistic as in section 3.1.

The adequacy of breakpoints to capture the nonlinearity plays an important role and hence needs to be determined. Initially, twenty breakpoints of angle of attack were considered with a step size of 1 degree as angle of attack ranges between –1 to 18 degrees. This yielded the regressor column size as twenty-two and the model structure determination was carried out. Incorporating apriori knowledge in the process of estimation is a well-known concept that can improve accuracy or save some computational efforts. The wind tunnel data for C_m exhibited linearity for angle of attacks ranging between –1 to 12 degrees. This prior information can be used in the estimation process to reduce the regressor column size, which in turn increases the computational speed. Hence, we considered angle of attack breakpoints as [-1, 13, 14, 15, 18] and performed the estimation. Further, we considered two more cases with angle of attack breakpoints at [-1, 13, 15,18] and [-1,13, 18] for a binary search analysis, to determine the adequacy of breakpoints leading to the adequate model structure. Table 2 summarizes the analysis to determine adequate breakpoints. We found that the 5 break point case is optimum in terms of R^2 and $Adj\ R^2$ statistics.

The nonlinear variation of estimated pitching moment coefficient is compared against its wind tunnel truth-value in Figure 4 for the five-breakpoint case. It is noted that the end point (angle of attack=18 deg) is captured better in case of nonlinear parameter estimation. Also, it can be noted from Table 2 that the five-breakpoint case is adequate as R^2 and $Adj\ R^2$ statistics are greater than those of the linear in parameter estimation case tabulated in Table 1.

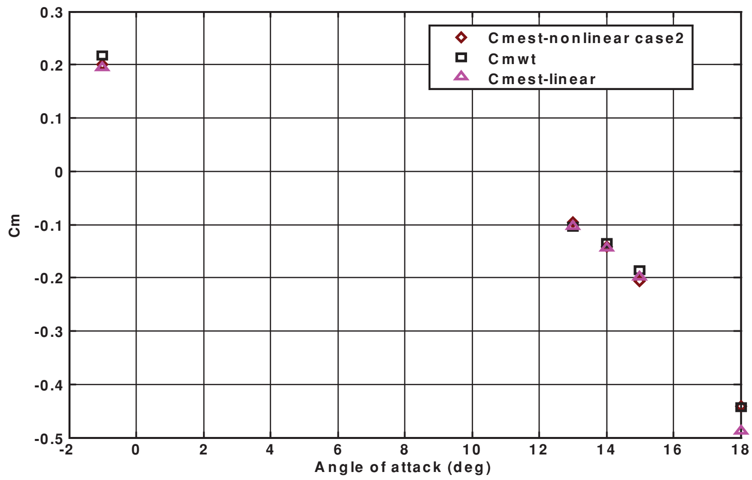

Figure 4. Comparisons of wind tunnel data with nonlinear and linear estimation

Figure 5 shows the model fit obtained (for the five breakpoint case) at every step compared with the true value, obviously the fit between model arrived at step 3 and true value is satisfactory as the model is adequate.

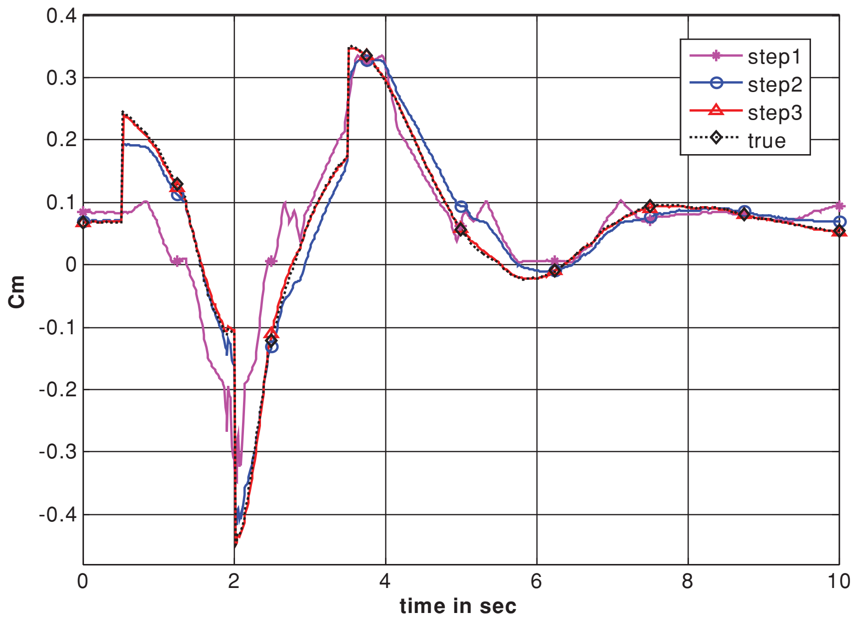

Figure 5. Stepwise model matched with true model (nonlinear estimation)

The choice of $F_{critical}$ was not very sensitive as applied to the method of nonlinear estimation whereas in the other technique where the parameters are linear, the stepwise regression was found to be sensitive to the choice of $F_{critical}$. If $F_{critical}$ were chosen to be 100 instead of 50 as was chosen then α^3 would not have found its presence in the model structure. It is noted that the R^2 and $Adj\ R^2$ statistics were found to be 0.9976 with exclusion of α^3 in the model, which is found to be lower when compared to 0.9985 as produced by the nonlinear estimation technique.

4. APPLICATION TO THE AERO DATABASE UPDATE OF HIGH PERFORMANCE FIGHTER AIRCRAFT

The technique is validated using the six-DOF simulated data of a transport aircraft under Section 3, it is important to test this technique with flight test data. Therefore, we consider a flight test data of a high performance aircraft to update the aero database. This aircraft is longitudinally unstable and hence it is augmented with full authority control laws. The longitudinal six-DOF simulation based on the nominal wind tunnel data shows differences in normal acceleration, pitch rate, angle of attack and elevator deflection when compared with corresponding flight responses. Hence it is necessary to update the aero-database in order to match the six-DOF simulation responses with the flight responses. The angle of attack is seen to vary from -1^0 to 9^0. It can be noted that the technique is validated using simulated data for angle of attack ranging -1^0 to 18^0. Whereas when it is to be tested for real flight data, initially we considered medium amplitude piloted stick inputs that could cover angle of attack ranging -1^0 to 9^0, whose results are reported here. When additional data at higher angles of attack is available we would validate the model fit. As the aircraft short period responses do not match, the total pitching moment is estimated using the following model, after determining the adequate structure as outlined earlier in this paper:

$$C_{m-f} = C_{mo-f} + C_{m-f}(\alpha) + C_{mq-f}\frac{q\bar{c}}{2v} + C_{m\delta e-f}\delta e \tag{13}$$

Noted that the C_m has joint dependency on angle of attack and Mach number. It is a common practice to consider Mach number constant and vary the angle of attack to generate the data for modeling C_m. Subsequently; different such segments obtained at various Mach numbers will be concatenated to get the dependency on Mach number. In this, paper we have reported a model for one flight segment where the Mach number is constant. The Mach dependency will have an additional term in (13). After the estimation is performed, the updated tables of coefficients are incorporated in the six-DOF simulations. The simulation is again performed and the resultant response match is shown in Figure 6.

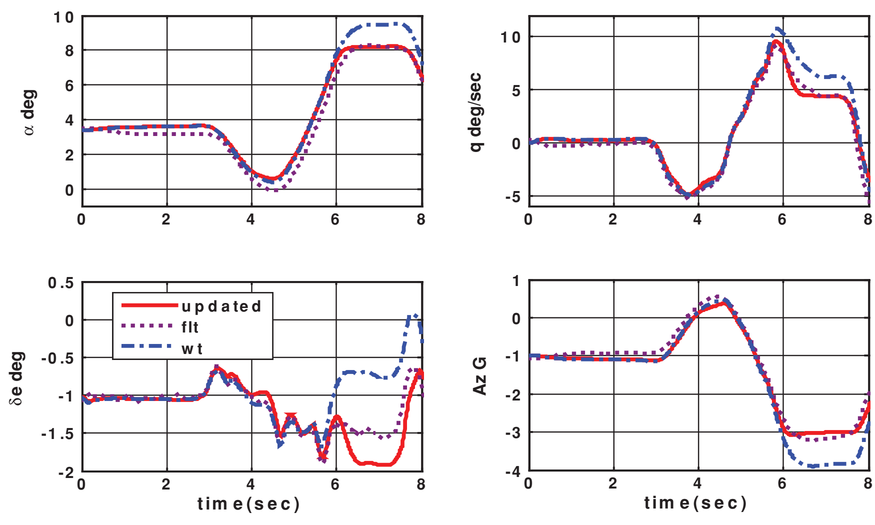

Figure 6. Time response comparison for model postulated in (13)

The updated six-DOF simulation responses give a better match with flight data than with the original wind tunnel based six-DOF simulations. It is important to find out which component in the application rule for the total pitching moment equation is causing this mismatch. This is a very important inference that will help to verify and refine the control laws, as the aircraft is longitudinally unstable. To accomplish this, a sensitivity analysis is performed. The wind-tunnel application rule for the total pitching moment coefficient in the six-DOF simulations takes the following form:

$$C_{m-wt} = C_{mo-wt} + C_{m-wt}(\alpha) + C_{mq-wt}\frac{q\bar{c}}{2v} + C_{m\delta e-wt}\delta e \tag{14}$$

where $C_{mq\text{-}wt}$ and $C_{m\delta e\text{-}wt}$ are constants for the considered flight condition.

In the sensitivity analysis, one component from (14) is removed at a time and is replaced by the corresponding updated/estimated component. It was found that changing C_{mo}, C_{mq}, $C_{m\delta e}$ did not improve the fit between six-DOF simulations and flight responses. The replacement of $C_{m\text{-}wt}(\alpha)$ with $C_{m\text{-}f}(\alpha)$ alone improved the above-mentioned fit. Hence the modified application rule for the pitching moment coefficient in the six-DOF simulations takes the following form:

$$C_{m-wt} = C_{mo-wt} + C_{m-f}(\alpha) + C_{mq-wt}\frac{q\bar{c}}{2v} + C_{m\delta e-wt}\delta e \tag{15}$$

where the suffixes '*f*' and '*wt*' denotes flight and wind-tunnel. The fit of six-DOF simulation responses with flight responses after incorporating the above equation is shown in Figure 7. The fit appears satisfactory.

It is desirable to perform one more level of validation after the database update. We choose another flight data segment that is close to the previous segment with respect to the flight conditions. The previously obtained estimates were validated using this segment. The match between the flight responses and six-DOF simulation responses before and after update is shown in Figure 8. The fit between flight and updated responses appears to be satisfactory.

In Figures 6-8, the fit for the elevator deflection is not good, however the updated responses are closer to the flight when compared with original responses obtained using wind tunnel data. This may be because, we updated only C_m and not the other longitudinal coefficients and also only one segment of data is used for the estimation. Hence, in future by concatenating various segments covering a wider range of angle of attack and by updating all the longitudinal coefficients we expect to overcome this problem. Also note that only the aero dynamic database is updated using this technique (wind tunnel to match flight). The control laws have not undergone any updates.

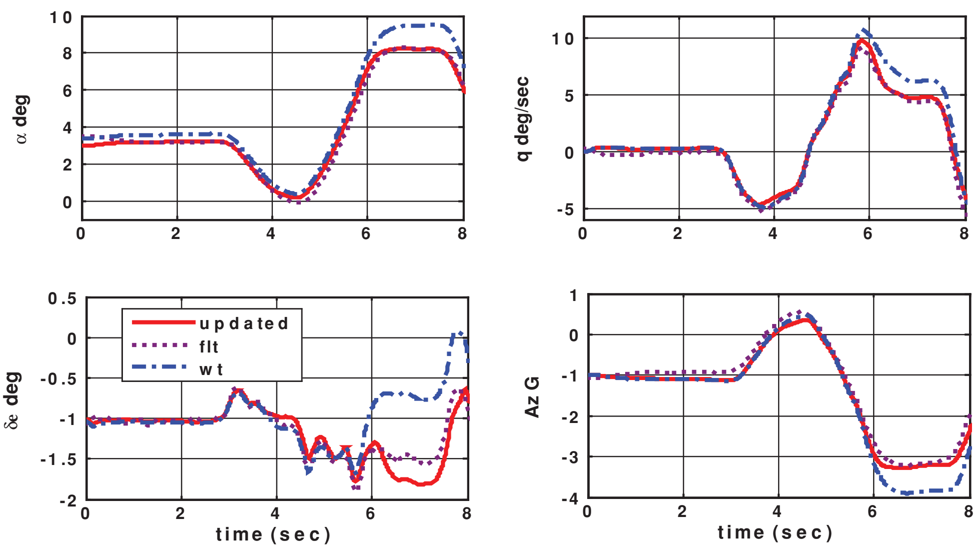

Figure 7. Time response comparison for model postulated in (15)

CONCLUSION

The paper presents the statistical measures required to arrive at an adequate nonlinear aerodynamic model structure by evaluating two different types of nonlinear models. The model structure determination of models, which are nonlinear in parameters, require less computational efforts when compared to models that are linear in parameters and nonlinear in independent variables. The results are validated for simulated data of a transport aircraft and the technique is applied to the aerodynamic database update of a high performance fighter aircraft.

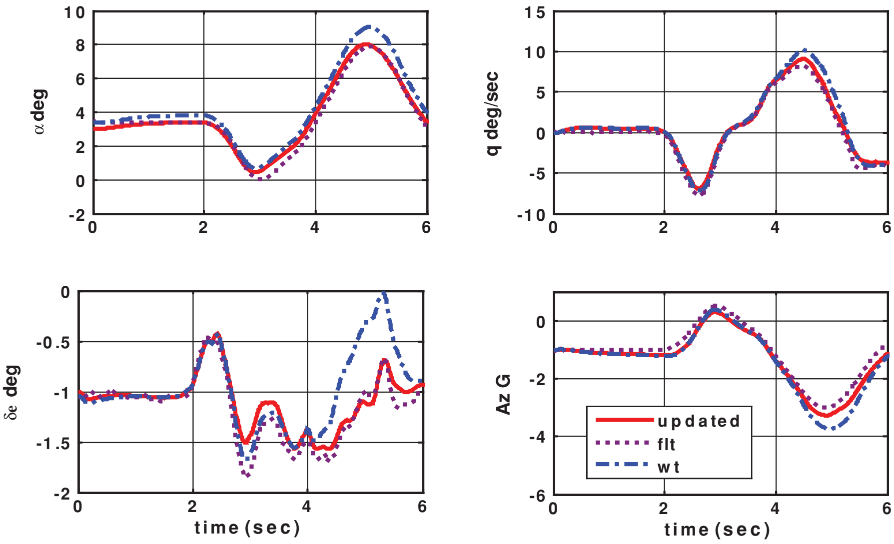

Figure 8. Validation of postulated in (15) with another segment

Table 1: Statistical measures for model structure determination of model being linear in parameters and nonlinear in response

step	Postulated model	*F*statistic: 1st significant variable in STEP 1 (F_j)	*F*statistic: New entry in current step F_k	R^2 statistic	*Adj* R^2 statistic
1	Start with $C_m = C_{m0}$	-	-	1.5e-35	0.0013
Set of independent variables defined are $\alpha, q\bar{c}/2v, \delta e, \alpha^2, \alpha^3, \alpha^4, \alpha^5$					
Partial Correlation for all the independent variables in descending order 0.7862(α^2), 0.7754(α^3), 0.7717 (α), 0.7579 (α^4), 0.7402(α^5), 0.31630(δe), 0.0285($q\bar{c}/2v$). α^2 has highest partial correlation and it is called the **1st significant variable.**					
2	$C_m = C_{m0} + C_{m\alpha^2}\alpha^2$			0.6181	0.6172
Partial Correlation for the remaining variables (descending) 0.8853 (δe), 3796($q\bar{c}/2v$), 0.1378(α^5), 0.1347 (α), 0.1024 (α^4), 0.0536(α^3)					
3	$C_m = C_{m0} + C_{m\alpha^2}\alpha^2 + Cm_{\delta e}\delta e$	1.6207e+004	6.1668e+003	0.9174	0.9171
Min(Fj,Fk)=min(6.1668e+003, 1.6207e+004)>Fcritical , variable added at step 3 is contributing significantly to the model fit. This completes one iteration and we loop back to examine partial correlation of remaining independent variables.					
Partial Correlation for the remaining variables (descending order) 0.7909($q\bar{c}/2v$), 0.3769 (α), 0.1233(α^3), 0.0429 (α^4), 0.0206(α^5)					
4	$C_m = C_{m0} + C_{m\alpha^2}\alpha^2 + Cm_{\delta e}\delta e + C_{mq}q\bar{c}/2v$	2.0260e+004	1.5093e+003	0.9691	0.9689

<table>
<tr><td colspan="6">Min(Fj,Fk)=min(2.0260e+004, 1.5093e+003)>Fcritical , variable added at step 4 is contributing significantly to the model fit. This Completes second iteration and loop back to examine partial correlation of remaining independent variables.</td></tr>
<tr><td colspan="6">Partial Correlation for the remaining variables in descending order 0.4568 (α), 0.2197(α^3), 0.1445 (α^4), 0.0849(α^5)</td></tr>
<tr><td>5</td><td>$C_m = C_{m0} + C_{m\alpha^2}\alpha^2 + Cm_{\delta e}\delta e + C_{mq}q\overline{c}/2v + C_{m\alpha}\alpha$</td><td>1.1799e+003</td><td>223.4498</td><td>0.9715</td><td>0.9713</td></tr>
<tr><td colspan="6">Min(Fj,Fk)=min(1.1799e+003, 223.4498)>Fcritical , variable added at step5 is contributing significantly to the model fit. This completes third iteration and loop back to examine partial correlation of remaining independent variables.</td></tr>
<tr><td colspan="6">Partial Correlation for the remaining variables in descending order 0.9565(α^5), 0.9334 (α^4), 0.8864(α^3)</td></tr>
<tr><td>6</td><td>$C_m = C_{m0} + C_{m\alpha^2}\alpha^2 + Cm_{\delta e}\delta e + C_{mq}q\overline{c}/2v + C_{m\alpha}\alpha + C_{m\alpha^5}\alpha^5$</td><td>988.4167</td><td>4.3788e+003</td><td>0.9976</td><td>0.9976</td></tr>
<tr><td colspan="6">Min(Fj,Fk)=min(988.4167, 4.3788e+003)>Fcritical , variable added at step6 is contributing significantly to the model fit. This completes fourth iteration and loop back to examine partial correlation of remaining independent variables.</td></tr>
<tr><td colspan="6">Partial Correlation for the remaining variables in descending order 0.5727(α^3), 0.5571 (α^4)</td></tr>
<tr><td>7</td><td>$C_m = C_{m0} + C_{m\alpha^2}\alpha^2 + Cm_{\delta e}\delta e + C_{mq}q\overline{c}/2v$
$+C_{m\alpha}\alpha + C_{m\alpha^5}\alpha^5 + C_{m\alpha^3}\alpha^3$</td><td>76.7699</td><td>213.1017</td><td>0.9984</td><td>0.9984</td></tr>
<tr><td colspan="6">Min(Fj,Fk)=min(76.7699, 213.1017)>Fcritical , variable added at step7 is contributing significantly to the model fit. This completes fifth iteration and loop back to examine partial correlation of remaining independent variables.</td></tr>
<tr><td colspan="6">Partial Correlation for the remaining variable 0.1239 (α^4)</td></tr>
<tr><td>8</td><td>$C_m = C_{m0} + C_{m\alpha^2}\alpha^2 + Cm_{\delta e}\delta e + C_{mq}q\overline{c}/2v$
$+C_{m\alpha}\alpha + C_{m\alpha^5}\alpha^5 + C_{m\alpha^3}\alpha^3 + C_{m\alpha^4}\alpha^4$</td><td>35.7490</td><td>18.7148</td><td>0.9984</td><td>0.9984</td></tr>
<tr><td colspan="6">Min(Fj,Fk)=min(35.7490, 18.7148)<Fcritical , variable added at step8 is not contributing significantly to the model fit. Hence model at step 7 is adequate.</td></tr>
</table>

Table 2 Quantitative Analysis to determine adequacy of breakpoints for the nonlinear estimation

S t	Postulated model	No of points	Partial correlation	Fstatistic	R^2 statistic	Adj R^2 statistic
1	$C_m = C_m(\alpha)$	20bpts	-	-	0.7360	0.7292
		5bpts	-	-	0.6855	0.6836
		3pbts	-	-	0.6774	0.6761
		4bpts	-	-	0.6855	0.6840
2	$C_m = C_m(\alpha) + C_{m_{\delta e}} \delta e$	20bpts	0.9379	620.91	0.9682	0.9674
		5bpts	0.9392	3.07e3	0.9629	0.9626
		3pbts	0.9365	5.88e3	0.9603	0.9601
		4bpts	0.9390	4.09e3	0.9628	0.9626
3	$C_m = C_m(\alpha) + C_{m_{\delta e}} \delta e + C_{mq} \frac{q\bar{c}}{2v}$	20bpts	0.9837	2.36e3	0.9989	0.9989
		5bpts	0.9800	7.79e3	**0.9985**	**0.9985**
		3pbts	0.9664	7.65e3	0.9974	0.9973
		4bpts	0.9791	9.32e3	0.9984	0.9984

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