Three-Dimensional Density Measurement of Supersonic and Axisymmetric Flow Field by Colored Grid Background Oriented Schlieren (CGBOS) Technique

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Abstract
The background oriented schlieren (BOS) technique is one of the visualization techniques that enable the quantitative measurement of density information in the flow field with very simple experimental setup. BOS requires only a background and a digital still camera and it can realize the quantitative measurement of density. In this report we propose the colored grid background image for BOS technique (CGBOS). The experiments were carried out in the 0.6 m × 0.6 m test section of supersonic wind tunnel at JAXA-ISAS. The measurement setup consisted of metal halide ramps, a colored grid background image and a digital still camera. A colored grid pattern was used as background image and density gradient in vertical and horizontal direction was obtained. The measurement result and prospect of CGBOS technique are reported.

1. INTRODUCTION

The background oriented schlieren (BOS) technique is based on a patent by Meier [1] and described by Richard [2] and Meier [3]. BOS technique enables the quantitative density measurement with very simple optical setup by computer-aided image analysis. In the past several years, BOS technique has been applied to various experiments—wind tunnel experiment [3–5], free flight experiment [5], free jet [5, 6], rotor blade tip vortex of full-scale helicopter [7], etc. The sensitivity and accuracy of BOS is examined by Goldhahn and Seume [6]. Recently Venkatakrishnan and Suriyanarayanan reported precise measurement of 3D density field of separated flow by BOS [8]. Most BOS techniques employ a monochromatic or colored random dot pattern as a background image. The displacement of the dot pattern is calculated by a cross-correlation algorithm commonly used in PIV (Particle Image Velocimetry) technique. On the other hand, a horizontal stripe was employed for the background image in synthetic schlieren [9, 10]. In these reports, the displacement of line pattern was calculated by taking the difference between two images in contrast to calculating the center position of lines directly with the finite-fringe analysis method in this report. CGBOS (Colored-Grid Background Oriented Schlieren) technique employing colored-grid pattern as a background is proposed and it is applied to the reconstruction of density in an axisymmetric flow field.

2. BACKGROUND ORIENTED SCHLIEREN TECHNIQUE

The principle of BOS is similar to the conventional schlieren technique, which exploits the bending of light caused by refractive index change corresponding to density change in the medium and both techniques are sensible to density gradient. Conventional schlieren technique is still important technique for shock geometry and high-speed frequency analysis, and the real time measurement, etc. Many quantitative schlieren techniques have been reported and recently a comparison of calibrated schlieren, rainbow schlieren and BOS is reported [11]. Detailed explanation of quantitative schlieren

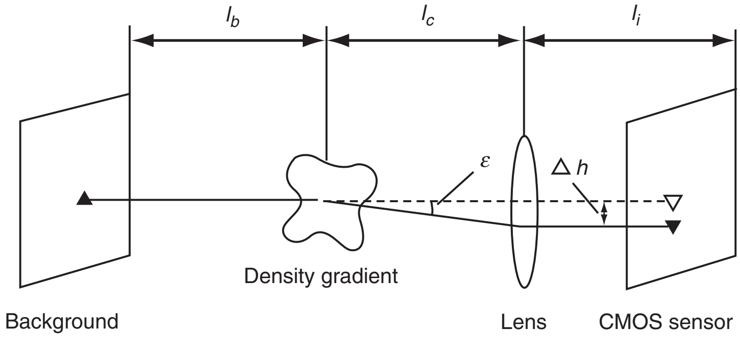

Figure 1. Optical setup of BOS measurement.

techniques are described in the paper. Conventional schlieren and quantitative schlieren employ many optical elements—pinhole, concave mirror, knife-edge or color filter, camera . . . etc. and calibration is required for quantitative measurement [11].

BOS requires only a background and a digital still camera and it can realize the quantitative measurement of density with no calibration. Figure 1 shows optical setup for BOS technique [5]. If there is a density change between the background and camera, the background image is captured at the CMOS sensor of the digital still camera with displacement Δh because of the refraction of the light passing through the density gradient as shown in figure 1 with a solid line. The relation between Δh and refractive index n is expressed as eqn. (1) where l_b denotes the distance from background to phase object, l_c the distance from phase object to camera, l_i the distance from camera lens to image sensor, f the focal length of camera, n the refractive index and ε deflection angle [2]. The relation between density ρ and refractive index n is given by the Gladstone–Dale equation expressed as eqn. (2), where G is the Gladstone–Dale constant. The integration of the spatial gradient of the refractive index along light passes can be obtained from eqn. (1) by measuring displacement Δh with image analysis. The density information can be also determined with eqn. (2).

$$\Delta h = \frac{l_b f}{l_b + l_c - f} \frac{1}{n_0} \int_{l_b - \Delta l_b}^{l_b + \Delta l_b} \frac{\partial n}{\partial r(x,y,z)} dl \tag{1}$$

$$\frac{n-1}{\rho} = G \tag{2}$$

3. EXPERIMENTS

The experiments were carried out in a supersonic wind tunnel at JAXA/ISAS which has 60 × 60 cm^2 test section. Schematic diagrams of a supersonic wind tunnel and test model are illustrated in Fig. 2 and Fig. 3 respectively. The measurement system consists of a metal halide lamp (continuous), a background and a digital still camera (EOS Kiss Digital X) which has a 3880 × 2690 pixels CMOS sensor. The background is colored grid pattern as shown in Fig. 4. In this paper colored grid background is composed of horizontal green stripe and vertical red stripe. The distances l_b and l_c are set to 710 mm and 5080 mm as shown in Fig. 5. The focal length of the camera f is 480 mm and shutter speed is 1/80 s. Thus the mean density field of supersonic flow is captured. Mach number of supersonic flow is set to 2.0. The experimental setup of CGBOS measurement is equivalent to Schardin's report [12, 13] as described in Ref. 11, however employing a digital camera and image processing for quantitative measurement is a new approach. Schardin introduced stripes as the background which called 'schlieren method no. 2' [12, 13], CGBOS employs colored grid to measure density gradients in horizontal and vertical directions from one exposed image in contrast to Schardin's work which employs monochromatic stripe-pattern for visualization.

Figure 6 shows CGBOS image taken through Mach 2.0 flow. The distortions of the background image along the shock wave and expansion wave can be recognized. The CGBOS image can be separated into green (horizontal) and red (vertical) stripe images by color information. The distortion

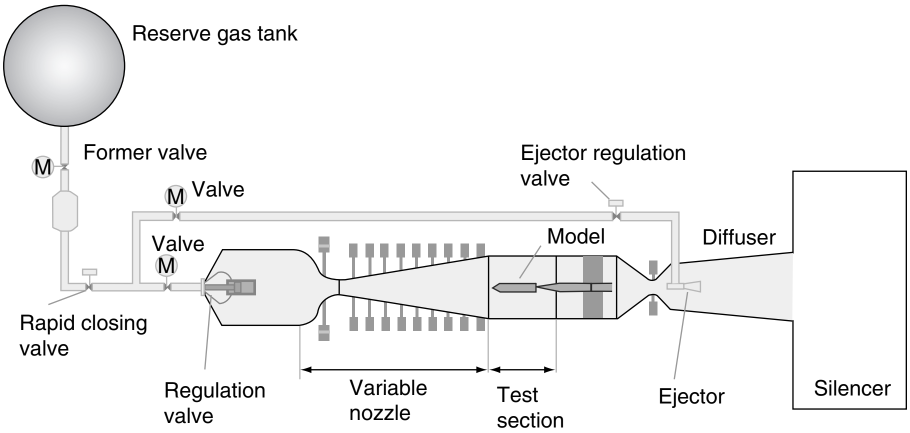

Figure 2. Supersonic wind tunnel at JAXA/ISAS.

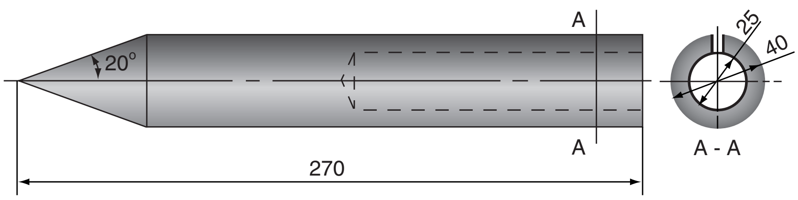

Figure 3. Test model (circular cone).

Figure 4. Colored grid background.

of the background image in the vertical direction is obtained from horizontal green stripes and in the horizontal distortion is obtained from vertical red stripes. Separated horizontal green-stripe and vertical red-stripe images around the tip of the model are shown in Fig. 7. The displacement in vertical and horizontal direction of each stripe pattern can be obtained with finite-fringe analysis technique of LICT measurement developed in our group [14–17]. Displacement of background can be measured in the center position of stripes thus displacement data can be obtained continuously in the stripe direction and we expect that higher resolution can be achieved compared with other BOS measurements using random dots. The gray-scale images of calculated displacement of Fig. 7 are shown in Fig. 8. These images indicate distribution of vertical and horizontal displacement of background image in 8 bits gray-scale. Black and white represent the shift in lower and upper direction for vertical displacement and in right and left direction for horizontal displacement. Left

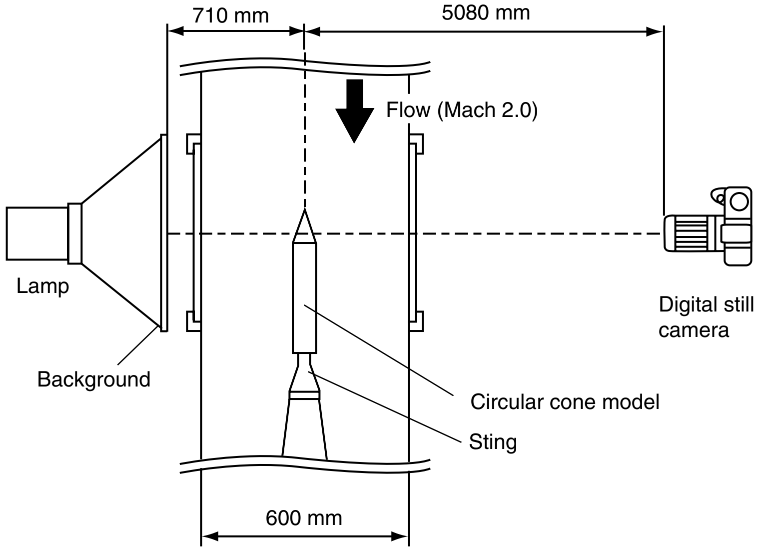

Figure 5. BOS setup.

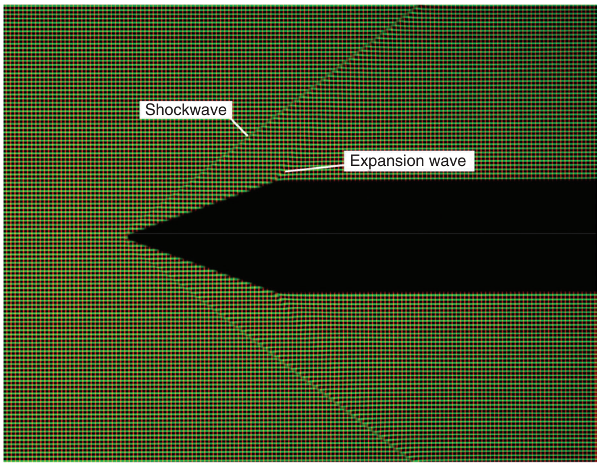

Figure 6. CGBOS image.

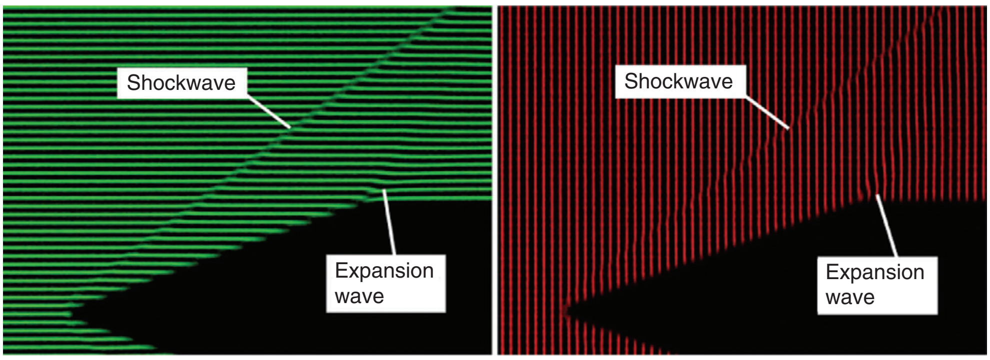

Figure 7. Separated images, horizontal green stripe (left) and vertical red stripe (right).

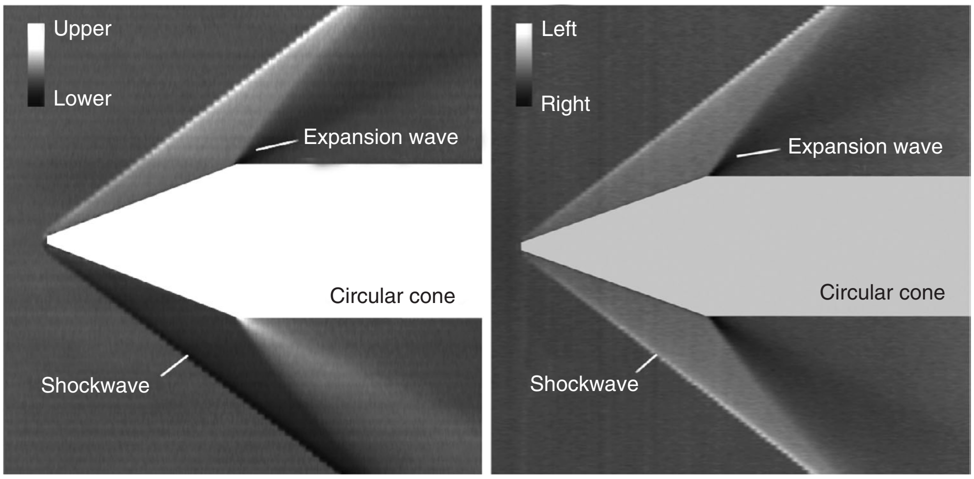

Figure 8. Gray-scale images of calculated displacement, vertical displacement of background obtained from horizontal green stripe (left) and horizontal displacement obtained from vertical red stripe shown in Fig. 7.

image of Fig. 8 represents the density gradient in the vertical direction and is equivalent to the conventional schlieren image taken with a horizontal knife-edge, and right image represents the density gradient in the horizontal direction and is equivalent to the conventional schlieren image taken with a vertical knife-edge. Both images in Fig. 8 agree well with conventional schlieren images shown in Fig. 9. Figure 10 is a comparison between the displacement images (displacement of background image in pixel) obtained with CGBOS (*a* and *c*) and pseudo images of displacements calculated from the density distribution around a cone model from cone table (*b* and *d*) [18]. Pseudo images are obtained by line-of-sight integration of the density gradient calculated from theoretical density distribution from cone table [18] and vertical and horizontal displacement of background image on the camera sensor is calculated with eqn (1). Good agreements between pseudo-displacement images and displacement images from CGBOS are achieved except that captured shock wave in CGBOS result seems to be thicker than pseudo displacement images and conventional schlieren images. The BOS techniques measure the flow phenomena with diverging light, therefore the captured image could contain blurs. Plots of displacement data on line A-A' in Fig. 10 are shown in Fig. 11. The left plot represents vertical displacement and right plot represents horizontal

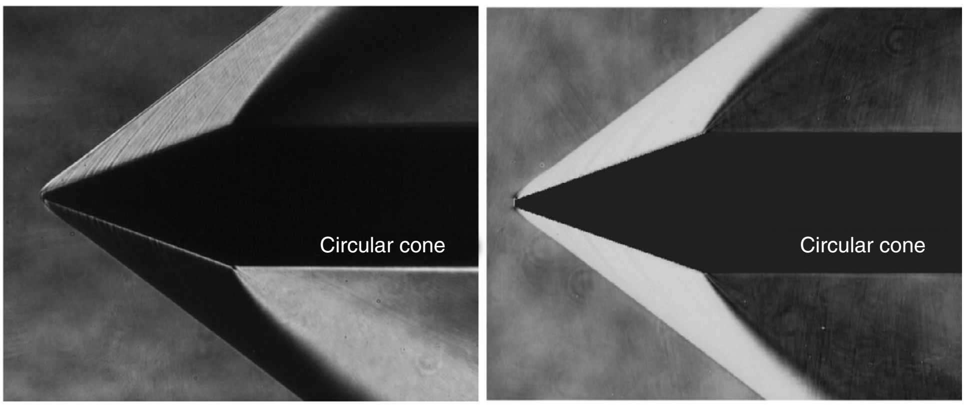

Figure 9. Conventional schlieren images, taken with horizontal knife-edge (left) and taken with vertical knife-edge (right).

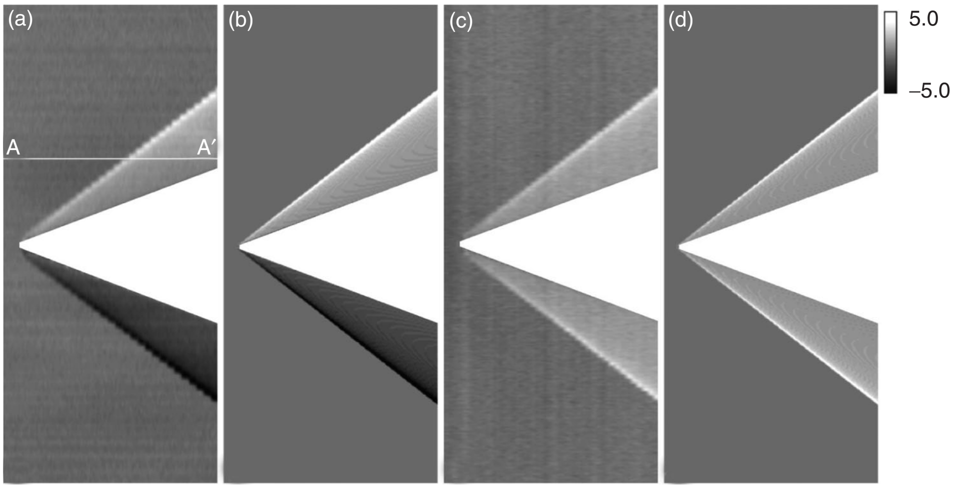

Figure 10. Comparison between CGBOS displacement image and pseudo-CGBOS image obtained with density distribution from cone table [18]; (a) Vertical displacement obtained from CGBOS (shown in Fig. 8), (b) Pseudo image of vertical displacement calculated from cone table, (c) Horizontal displacement from CGBOS (shown in Fig. 8), (d) Pseudo image of horizontal displacement calculated from contable [18].

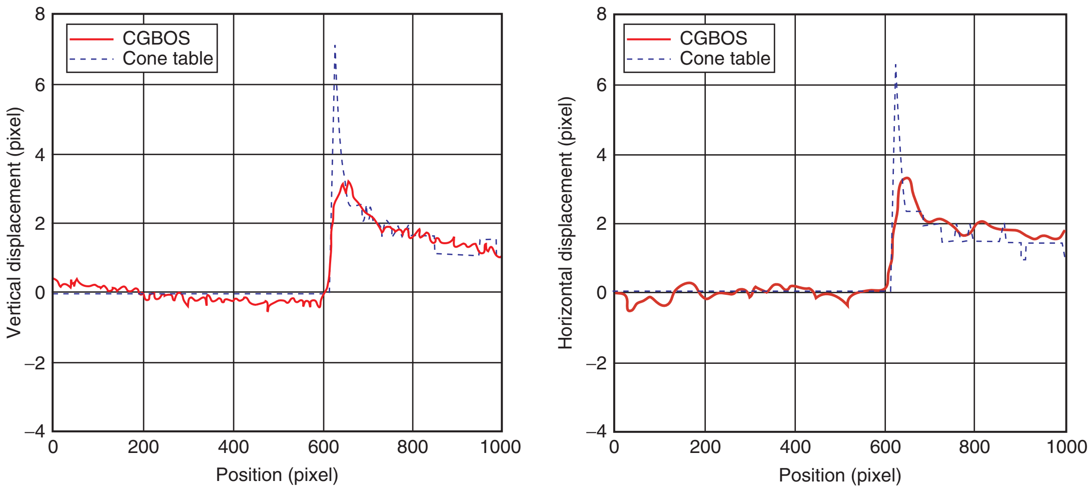

Figure 11. Plot of displacements on line A-A' in Fig. 10.

displacement respectively. A good agreement can be recognized for both vertical and horizontal displacement. It is seems that shock wave is captured thick as mentioned above, however density information can be measured accurately with CGBOS measurement.

4. RECONSTRUCTION OF DENSITY FIELD

Three-dimensional density distribution of supersonic flow field can be reconstructed from projection data with CT (Computed Tomography) technique. The integration of the gradient of refractive index for observation area can be obtained with BOS technique as shown in eqn (1). For an axisymmetric flow density distribution can be obtained with Abel transformation or Fourier transform based technique. Venkatakrishnan *et al*. reconstructed 3D density distribution with one projection data for axisymmetric flow on the assumption that infinite number of projection is obtained [4, 8]. This paper describes reconstruction result with ART (Algebraic Reconstruction Technique) [19] on the assumption that same projection data of axisymmetric flow field is obtained from 36 projection angles from 0° to 175° with 5° intervals. Thus 3D density distribution is reconstructed from one projection data obtained from experiment. Reconstruction from 36 projections is the same procedure with our previous report [20] that describes the reconstruction of asymmetric flow field from multi-angle projection data obtained with CGBOS technique. This paper examines the accuracy of CT reconstruction with CGBOS technique by comparing with theoretical value from cone tables [18].

ART is one of the iterative reconstruction methods and consists of assuming that the cross section consists of an array of unknowns, and then setting up algebraic equations for unknowns in terms of the measured projection data [19]. ART is much simpler than FBP (Filtered Back Projection) method which is the transform-based algorithm [21]. For FBP a large number of projections are required for higher accuracy in reconstruction. In addition the flow field around a cone model have to be reconstructed from incomplete projection data due to the obstruction caused by the model. As described in Ref 22, obstructed part has to be filtered or interpolated for FBP algorithm to avoid the numerical artifacts. However it is not clear that the interpolation is physically correct or not. In this report flow field around a circular cone is reconstructed from projection data on visible part by ART and the obstructed part does not contribute to the reconstruction. The number of projections could be limited and the projection data contains incompleteness due to the existence of models in the flow for the realistic experiments. In the instance ART is more amenable than FBP. A comparison of FBP and ART reconstruction of the same experimental results and the reconstruction of high-speed and unsteady flow field around an object with ART related to LICT measurement can be found in Ref. 23.

The iteration process of ART on x–y plane (shown in Fig. 12) can be described as eqn. (3) where f^i denotes guess at i step, P projection data of experiment, R^i pseudo-projection calculated from f^i distribution, C the number of pixels in a projection line, X position in projection plane and θ projection angle. For the reconstruction of 3D distribution of f^n (x, y, z), initial guess f^0 (= 0) is applied firstly and pseudo-projection R^0 is calculated with projection of initial guess f^0. And then next guess f^1 is calculated by eqn. (3) which revising guess by evaluation of a difference between measured projection P and R^0. Applying this process to all projection angles (from 0° to 175°), one step of ART iteration on an x–y plane is completed. In this report 30 iteration steps is applied for an x–y plane and 3D distribution of f^n (x, y, z) is obtained by the collection of 2D reconstructions of f^n (x, y).

BOS techniques measure the integration of local refractive index gradient along the right pass as eqn. (1), the integrated gradient of refractive index in X direction on projection plane can be obtained from vertical displacement of the background image and the integrated gradient of n in x and y directions ($\partial n/\partial x$ and $\partial n/\partial y$) can be obtained with the trigonometric relation between x, y and X as shown in Fig. 12. The integrated gradient of n in the z direction ($\partial n/\partial z$) can be obtained from horizontal

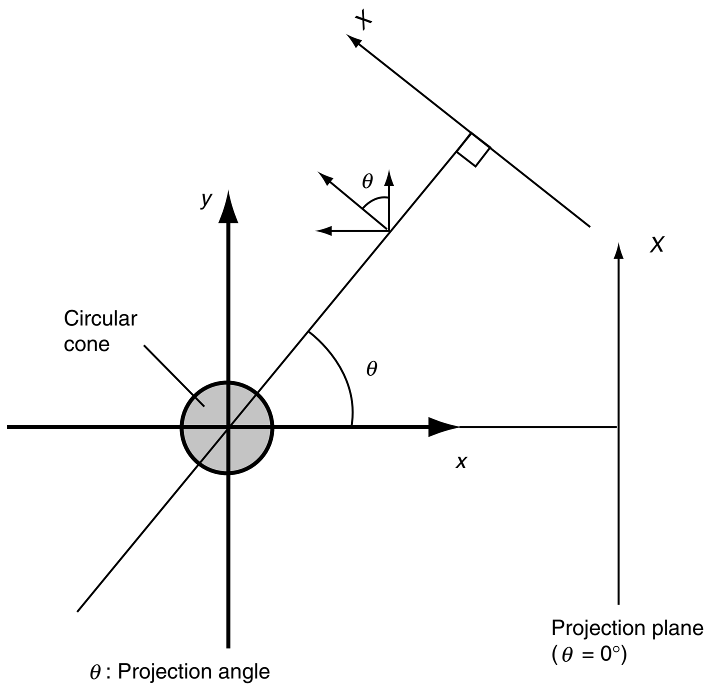

Figure 12. Projection angle.

displacement of the background image. Thus ART is applied to 3 components of x, y and z individually. After the reconstruction we can obtain local gradient of refractive index $\partial n/\partial x$, $\partial n/\partial y$ and $\partial n/\partial z$ in a flow field. Three-dimensional distribution of n is determined by solving the Poisson equation expressed in eqn. (4) using Successive Over Relaxation (SOR) method. Where S is a source term and it is obtained by calculating the gradient of reconstructed $\partial n/\partial x$, $\partial n/\partial y$ and $\partial n/\partial z$ with finite difference method. Finally 3D normalized-density distribution is obtained with the relation between ρ and n as expressed in eqn. (5).

$$f^{i+1}(x,y) = f^{i}(x,y) + \frac{P(X,\theta) - R^{i}(X,\theta)}{C(X,\theta)} \tag{3}$$

$$\frac{\partial^2 n}{\partial x^2} + \frac{\partial^2 n}{\partial y^2} + \frac{\partial^2 n}{\partial z^2} = S \tag{4}$$

$$\frac{\rho}{\rho_0} = \frac{n-1}{n_0-1} \tag{5}$$

Reconstructed density distribution on a central plane crossing the central axis of circular cone model is illustrated in Fig. 13. Density distribution in a flow field is normalized with free stream density and shown in contour. Freestream density (ρ_0) is calculated from isentropic relations together with chamber pressure and temperature where these values are known in experiments. Normalized density distribution near the surface of circular cone model is also illustrated. Figure 14 shows reconstructed 3D density distribution of whole flow field. Shock wave and expansion wave are captured quantitatively. Figure 15 is plots of normalized density for theoretical value from cone table [18] and CGBOS result. The variation of normalized density at 12 mm from the tip of circular cone in a streamwise direction is shown in diamond. Vertical axis is angle from the central axis and plot is for the range from 20° (model surface) to 37.8° (shock wave angle for Mach 2.0 flow with isentropic relations). The maximum relative error between cone table and CGBOS is about 4.0% in Fig. 15. From Fig. 13, density distributions behind shock wave seems to agree

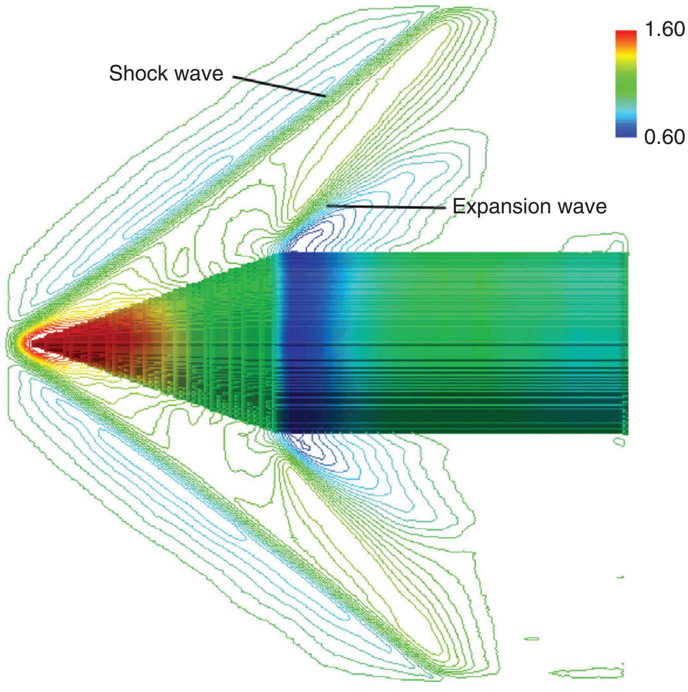

Figure 13. Reconstructed density distribution (ρ/ρ_0): contour on a central plane and pseudo-color near the surface of circular cone.

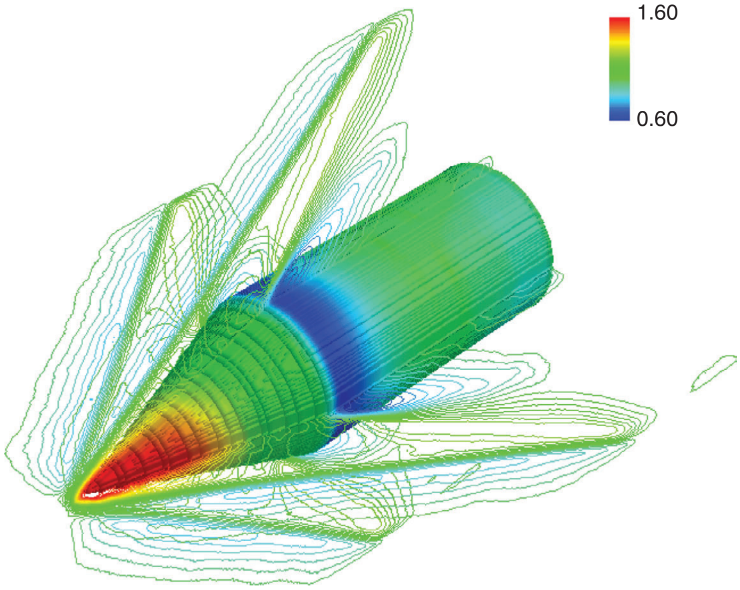

Figure 14. Reconstructed 3D density field.

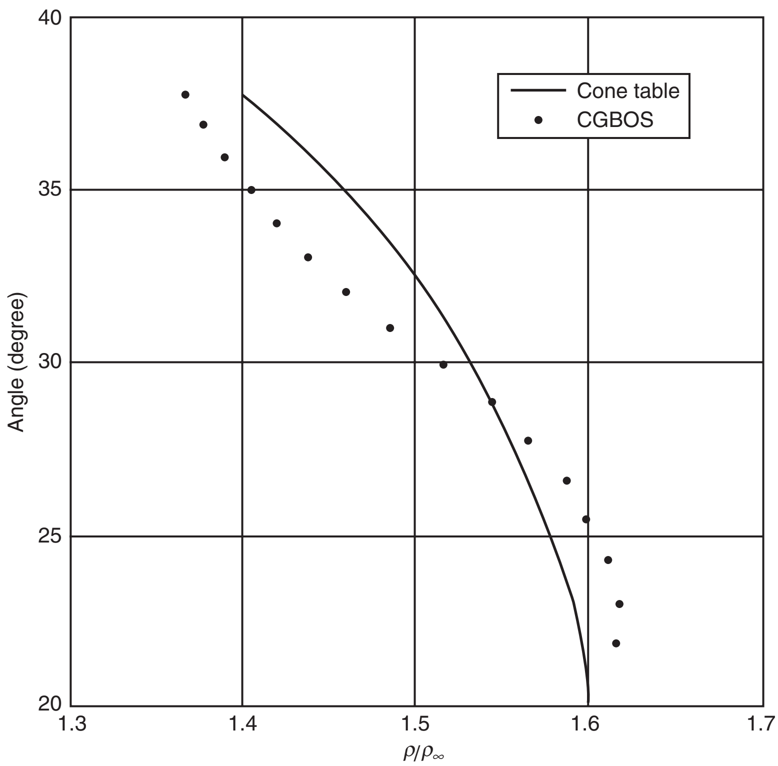

Figure 15. Comparison of normalized density behind the shock.

with theoretical value calculated with isentropic relation around the tip of cone, however errors seem to become larger around the junction of cone and cylinder. Low-density region in front of shock wave is also confirmed. The vertical and horizontal displacement data obtained with CGBOS measurement agree with pseudo-CGBOS image in Fig. 10 and Fig. 11, thus these errors could be caused by the iteration process for solving the Poisson equation with SOR method mentioned above.

5. CONCLUSION

The CGBOS technique using colored-grid background is proposed and applied to axisymmetric flow filed. The captured colored-grid background image was separated into horizontal and vertical stripes based on color information and displacements of separated stripes are calculated with the finite-fringe analysis technique. The resultant images of displacement in vertical and horizontal directions obtained from one exposure supply good agreement with conventional schlieren image. Pseudo images of displacements in vertical and horizontal direction of background are also agreed well with resultant displacement image of CGBOS measurement.

Three-dimensional distributions of density are reconstructed by ART, and the density information in axisymmetric flow field is obtained quantitatively. The agreement between the resultant density distributions obtained from CGBOS technique and theoretical value seems to be good. Some improvements in integration process may be required to obtain more accurate results.

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REFERENCES

[1] Meier G. E. A., Hintergrund Schlierenverfahren *Deutsche Patentschrift*, 1999, DE 19942856 B4.

[2] Richard H. and Raffel M., *Principle and applications of the background oriented schlieren (BOS) method, Meas. Sci. Technol.*, 2001, 12, 1576–85.

[3] Meier G., Computerized background-oriented schlieren, *Exp. Fluids*, 2002, 33, 181–7.

[4] Venkatakrishnan L. and Meier G. E. A., Density measurements using the background oriented schlieren technique Exp. Fluids, 2004, 37, 237–47.

[5] Leopold F., Simon J., Gruppi D. and Schäfer H. J., Recent improvements of the background oriented schlieren technique (BOS) by using a colored background *Proc. 12th Int. Symp. on Flow Visualization (German Aerospace Center (DLR), Go ttingen, Germany)*, 2006, ISFV12–3.4.

[6] Goldhahn E. and Seume J., The background oriented schlieren technique: sensitivity, accuracy, resolution and application to a three-dimensional density field, *Exp. Fluids*, 2007, 43 241–9.

[7] Kindler K., Goldhahn E., Leopold F. and Raffel M., Recent developments in background oriented schlieren methods for rotor blade tip vortex measurements, *Exp. Fluids*, 2007, 43 233–40.

[8] Venkatakrishnan L. and Suriyanarayanan P., Density field of supersonic separated flow past an after body nozzle using tomographic reconstruction of BOS data, *Exp. Fluids*, 2009, 47, 463–73.

[9] Dalziel S. B., Hughes G. O. and Sutherland B. R., Whole-field density measurements by 'synthetic schlieren', *Exp. Fluids*, 2000, 28, 322–35.

[10] Onu K., Flynn M. R. and Sutherland B. R., Schlieren measurement of axisymmetric internal wave amplitudes, *Exp. Fluids*, 2003, 35, 24–31.

[11] Hargather M. J., Settles G. S., A comparison of three quantitative schlieren techniques, *Optics and Lasers in Engineering*, 2012, 50, 8–17.

[12] Schardin H., Die Schlierenverfahren und ihre Anwendungen, *Ergebnisse der Exakten Naturwissenschaften*, 1942, 20, 303–439. (English translation, Schlieren methods and their applications, NASA TT F-12731, 1970).

[13] Settles G. S., *Schlieren and shadowgraph techniques*, Springer, Berlin Heidelberg New York, 2001, p 86.

[14] Honma H., Ishihara M., Yoshimura T., Maeno K. and Morioka T., Interferometric CT measurement of three-dimensional flow phenomena on shock waves and vortices discharged from open ends, *Shock Waves*, 2003, 13, 179–90.

[15] Maeno K., Kaneta T., Morioka T. and Honma H., Pseudo schlieren CT measurement of three-dimensional flow phenomena on shock waves and vortices discharged from open ends, *Shock Waves*, 2005, 14, 239–49.

[16] Ota M., Koga T. and Maeno K., Laser interferometric CT measurement of the unsteady supersonic shock-vortex flow field discharging from two parallel and cylindrical *nozzles, Meas. Sci. Technol.*, 2006, 17, 2066–71.

[17] Ota M., Inage T. and Maeno K., An extension of laser-interferometric CT measurement to unsteady shock waves and 3D flow around a columnar object *Flow Meas. Instrum.*, 2007, 18, 295–300.

[18] Sims J. L., Tables for supersonic flow around right circular cones at zero angle of attack, NASA SP-3004.

[19] Kak A. C. and Slaney M., *Principle of Computerized Tomographic Imaging*, (New York: IEEE), 1988.

[20] Ota M., Hamada K., Kato H. and Maeno K., Computed-tomographic density measurement of supersonic flow field by colored-grid background oriented schlieren (CGBOS) technique, *Meas. Sci. Technol.*, 2011, 22, 104011.

[21] Shepp A. L. and Logan F. B. The Fourier reconstruction of a head section, *IEEE Trans. Nucl. Sci.*, Ns-21, 21–43.

[22] Sourgen F., Leopold F., Klatt D., Reconstruction of the density field using the Colored Background Oriented Schlieren Technique (BOS), *Optics and Lasers in Engineering*, 2012, 50, 29–38.

[23] Ota M., Udagawa S., Inage T., Maeno K., Interferometric measurement in shock tube experiments, *Interferometry-Research and Applications in Science and Technology*, InTech, 2012, 225–235.