A Simple Feedback Control Strategy for Controlling the Axial Compressor Surge

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ABSTRACT
This paper presents the results of a feedback technique for controlling low frequency surge oscillations in an axial compressor. To simulate surge a hybrid model is developed, by coupling the Moore – Greitzer potential flow model and the Navier – Stokes solution, to result in more realistic flow predictions. This hybrid model incorporates the viscous effect on the flow behaviour, which is neglected in the Moore – Greitzer model. The present model is shown to capture the existence of rotating stall disturbances in the combustor during a typical classic surge cycle as has been reported in the experiments. After modelling the surge we propose a steady feedback control scheme to minimise the adverse effect of the surge oscillations on the combustor operation. The results for a four hole and six hole feedback control mechanisms are reported and it is seen that the existing high frequency fluctuations in the unsteady variation of plenum pressure before control are virtually eliminated in the presence of a steady feedback control. Due to decrease in the frequency of plenum pressure variations in the presence of feedback control, the variations have now become smoother and are seen to be relatively more stabilised. It is also shown that the steady feedback control is effective in reducing both the hysteresis and the energy loss from the flow thereby the work output from the turbine is expected to improve which in turn can lead to greater cycle efficiency. The present study also reveals that by using six hole feedback mechanism complete recovery from the adverse pressure gradient is possible, but at the expense of a reduced pressure delivery by the compressor. It is expected that the scheme proposed here is relatively simple to implement compared to the many other control schemes available today.

1. INTRODUCTION

The operability of compression systems is limited at low mass flow rates by fluid dynamic instabilities leading to rotating stall or surge. Compressor surge and rotating stall are primary design constraints which effectively reduce the engine performance and consume a major fraction of the engine development program. These unsteady aerodynamic instabilities can lead to large penalties in the performance because they are difficult to predict during the design stage. Focus of the research on compressor flow instabilities has been mainly on suppressing surge and rotating stall to extend the stable operating range of the compression system. By proper modelling and control of these instabilities, compressor performance can be improved.

Most of the research to date on this topic has focused on either the performance model proposed by Moore and Greitzer [1] which is valid only for the flow inside the compressor or on the full Navier – Stokes solution for the compressor and the flow passage connecting the compressor to the combustor. The latter is computationally expensive due to the simulation of unsteady flow across the rotating compressor blades. Hence a hybrid approach has been adopted in the present work whereby the Moore – Greitzer model [1, 2] is used to analyse the flow through the compressor and the effect of viscosity on compressor flow dynamics is analysed by numerically solving the Navier – Stokes equation in a flow passage considered between the compressor and the combustion chamber. Then a simple control strategy has been adopted to control surge in this flow passage to minimise its adverse effect on the operation of the combustion chamber. The Moore – Greitzer theory models the compressor flow as flow through a cylindrical duct and treats the blade rows as actuator disks. In the present work, to model the viscous effects on the compressor flow, a straight cylindrical duct is purposely chosen in order to be consistent with the Moore – Greitzer model as will be clear from the subsequent sections.

Betchov and Criminale [3] have defined stability as the quality of being immune to small disturbances. Analysis of the stability of simple compression systems has been carried out by Emmons et al. [4], Taylor [5], Stenning [6], and many others. The general result that emerges is that the system will be dynamically unstable near the peak of the pressure rise/mass flow characteristic at some slightly positively sloped operating point [7]. Greitzer [8] developed a non – linear model to predict the transient response of a compression system subsequent to a perturbation from steady operating conditions. Greitzer asserted that there is an important non – dimensional parameter '*B*' that determines which mode of compressor instability – rotating stall or surge, will be encountered at the stall line depending on whether its value is below or above critical. As per Greitzer [8, 9] two distinct surge types can be seen, one known as classic surge where the flow will be pulsating without any reversal and the other in which there is significant flow reversal during part of the surge cycle that is identified to be deep surge. Based on the work of Moore [10], Moore and Greitzer developed an approximate theory [1, 2] capable of predicting the post – stall transients in multistage axial compression systems. This model explains the coupling between the rotating stall like and surge like motion and gives the nature of the operating point motion during a transient stall phenomenon. In the case of classic surge the blade passes in and out of the stalled flow regime which induces vibration in blades as discussed by Horlock [11] and Pinsley et al. [12]. Surge which behaves as global system instability can trigger combustion instability/oscillations [7]. So it is highly important to control surge if operation of the machine has to be sustained. Many complex surge control strategies have been reported by the works of Banaszuk and Krener [13], Giarre et al. [14], and Shehata et al. [15]. Researches [16, 17] have reported that low frequency flow oscillations occurring in rocket engines could be controlled by employing feedback techniques. Thus it appears that the compressor surge which has low frequency [7] may also be controlled by utilising feedback control strategies. If this becomes realisable then the combustor could be relieved from the risk combustion instability. Thus the aim of the present paper is to propose a control scheme which is simple yet effective in controlling the surge with a minimum of complexity so that it can be easily implemented.

The study presented in this paper involves three parts. First, the Moore – Greitzer model [1] is solved for the compressor for suitable values of input conditions and the surge flow through the compressor is obtained. Second, using this flow as a boundary condition the incompressible Navier – Stokes equation is solved, in the flow passage between the compressor and the combustion chamber, to simulate the viscous surge behaviour. Third, a simple feedback surge control technique of simultaneous suction and injection of mass flow is employed in this domain to minimise the adverse effect of the surge flow oscillation on the combustion chamber operation.

2. MODELLING

Several dynamic models for the unstable operation of the compression systems have been proposed but the model of Moore and Greitzer [1, 2] stands out unique in the sense that the rotating stall amplitude is included as a state and not manifested as a pressure drop which is the case in other models [18, 19, 20]. The lower order model of Moore and Greitzer captures the post – stall transients of a low speed axial compressor – plenum – throttle system. The lower order refers to the simplicity of the model which describes the compression system behaviour in three states in spite of the complex fluid dynamic phenomena that it models. Fig. 1 [21] shows the schematic of the model compression system used in this study. The assumptions that are involved in this model are that inviscid and irrotational flow having no radial variation enters the compressor, a large hub to tip radius ratio so that a 2D description seems

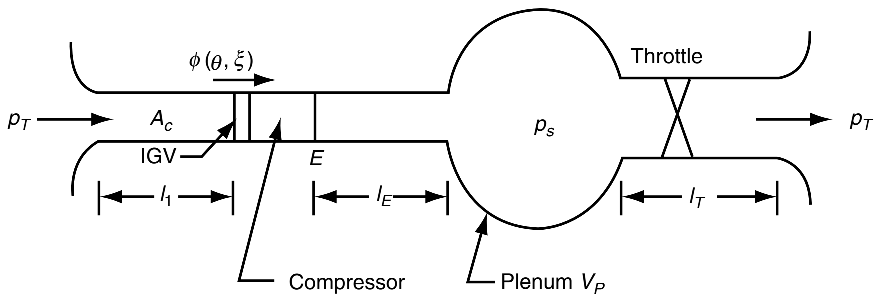

Figure 1. Schematic of the compressor showing non – dimensional lengths [21].

to be reasonable, incompressible flow on account of having low Mach number, compressible plenum gas with uniform static pressure (but unsteady), low pressure rises compared to the ambient conditions, constant rotor speed, and short throttle duct.

In the model for pursuing the studies, an ideal form of a compressor performance curve is needed in a sense that the performance of the compressor in the absence of any of the disturbances has to be known. In other words a performance curve, which would hypothetically be measured for a compressor with the rotating stall and surge absent, is required. In [22, 23] it has been argued that the characteristic curve is typically a smooth S – shaped curve and, hence, a physically realistic choice would be a simple cubic curve as shown in Fig. 2.

The expression for the curve [1] is as given below

$$\psi(\phi) = \psi_{c0} + H\left[1 + \frac{3}{2}\left(\frac{\phi}{W} - 1\right) - \frac{1}{2}\left(\frac{\phi}{W} - 1\right)^3\right] \tag{1}$$

The non – dimensional quantities used in ‘eqn. (1)’ are [1]

$$\Psi = \frac{p_s - p_T}{\rho^{U^2}} \tag{2}$$

$$\Phi = \frac{C_x}{U} \tag{3}$$

$$\varepsilon = \frac{Ut}{R}. \tag{4}$$

The three ordinary differential equations of the model [1] arises from a Galerkin approximation of the local momentum balance, the annulus averaged momentum balance and the mass balance of the plenum. These equations are

$$\frac{d\Psi}{d\varepsilon} = \frac{W/H}{4B^2}\left[\frac{\Phi}{W} - \frac{1}{W}\sqrt{\frac{2\Psi}{K_T}}\right]\frac{H}{l_e} \tag{5}$$

where $B = \frac{U}{2a_s}\sqrt{\frac{V_P}{A_c L_c}}$

$$\frac{d\Phi}{d\varepsilon} = \left[-\frac{\Psi - \psi_{c0}}{H} + 1 + \frac{3}{2}\left(\frac{\Phi}{W} - 1\right)\left(1 - \frac{1}{2}J\right) - \frac{1}{2}\left(\frac{\Phi}{W} - 1\right)^3\right]\frac{H}{l_c} \tag{6}$$

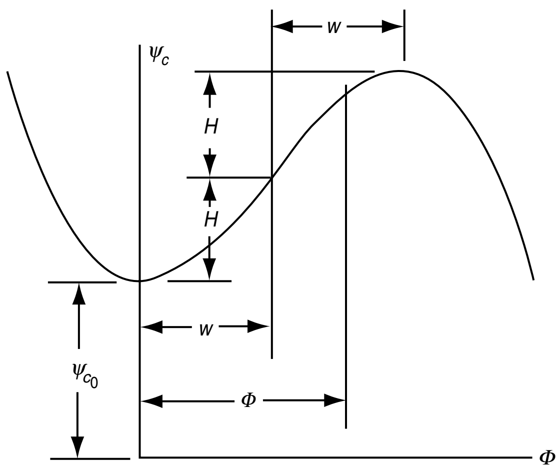

Figure 2. Cubic axisymmetric characteristic of Moore and Greitzer [1].

$$\frac{dJ}{d\varepsilon} = J\left[1-\left(\frac{\Phi}{W}-1\right)^2-\frac{1}{4}J\right]\frac{3aH}{(1+ma)W} \tag{7}$$

The derivation of these governing equations is not provided here in order to save space, and further details can be found in [1]. The schematic of the model compression system used in the present hybrid approach is as shown in Fig. 3. The incoming fluid enters the compressor through the inlet duct, reaches the combustor (or plenum) and moves out of the system through the exit throttle. The duct part located between the Moore – Greitzer compressor duct and the plenum is the computational domain for this work where the Navier – Stokes equations are solved. The mass flow solution of the Moore – Greitzer model [1] is supplied as the inlet boundary condition to this duct and an attempt is made to capture surge oscillations in this domain.

The Moore – Greitzer model equations (5), (6) and (7) have been solved by a MATLAB™ R2009a code using the inbuilt function 'ode 45', which uses a variable step Runge – Kutta Method to solve the ordinary differential equations numerically. The initial conditions used for solving the equations are

$$\Psi(0) = 0.66 \tag{8}$$

$$\Phi(0) = 0.5 \tag{9}$$

$$J(0) = 0.0004 \tag{10}$$

These conditions are obtained in the following manner.

Figure 4 shows the cubic axisymmetric characteristic used in the Moore – Greitzer model [1] to perform calculations for the post – stall transients. The curve shows a plot of the non – dimensional quantities of plenum pressure rise versus the compressor mass flow for a constant rotor speed. The throttle curve passing through the peak point of the ideal cubic compressor curve is also shown. The stall line marks the limit of the stable operation of the compressor. The stable operating point corresponds to the point of intersection of the cubic compressor curve and the throttle curve. The instability sets in when the operating point reaches the stall line and is represented by point B in Fig. 4. Thus the values of the non – dimensional plenum pressure Ψ and the annulus averaged compressor mass flow Φ corresponding to point B are chosen as the initial conditions to solve the Moore – Greitzer equations. At these conditions a level of circumferential non uniformity J is imposed on the compressor flow and the initial conditions of the system of equations to be solved are (taken from [1]):

The value of parameters and constants appearing in the equations used are provided in Table 1, following [1, 2].

H and W are the parameters appearing in the cubic axisymmetric characteristic that is shown in Fig. 2, a is a parameter that accounts for the inertial effects of flow through the compressor, m is a measure of

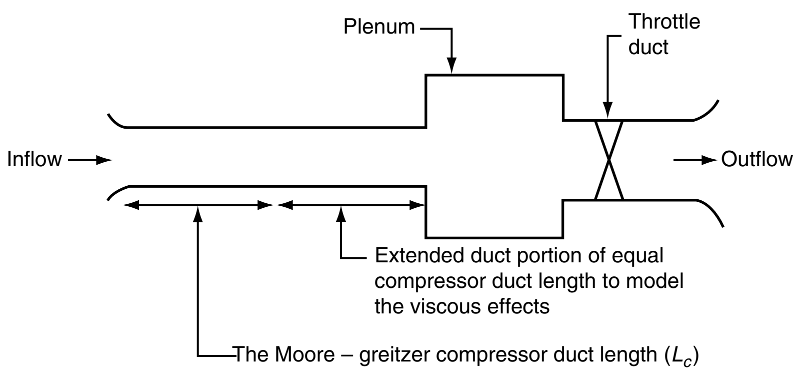

Figure 3. Schematic of the model compression system for the hybrid approach.

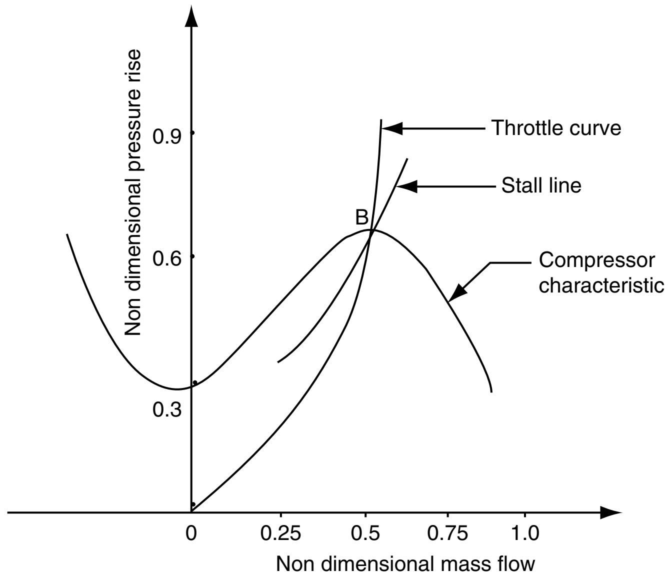

Figure 4. Compressor performance characteristic [1].

Table 1. Parametric values for the Moore - Greitzer model

Parameters	**Values**
H	0.18
W	0.25
a	2
m	1.75
B	1.0
l_c	8.0
ψ_{c0}	0.3
K_T	5.5

the duct length l_E relative to l_I in Fig. 1. m = 2 refers to a long enough exit duct, l_E and m = 1 would refer to a very short one. B is the familiar Greitzer parameter, l_c is the effective flow passage length for the system in Fig. 1, ψ_{c0} is the non – dimensionalised pressure rise through the compressor when the flow is zero, and K_T is the constant throttle coefficient.

The solutions of the Moore – Greitzer equations with the above mentioned initial conditions and parametric values are shown in Fig. 5. The solution shows a classic surge behaviour in which there is no reverse flow. This is evident from the value of axial flow coefficient, which always remains positive. From the temporal variation of the square of the disturbance amplitude (J) it is clear that the disturbance in compressor mass flow grows and decays periodically indicating the development and decay of rotating stall like circumferential flow distortions during a part of the classic surge cycle.

To make the flow computations in the duct faster, the time dependent solution for the axial flow coefficient and the non – dimensional plenum pressure obtained from the Moore – Greitzer model are expressed as a closed form solution through a Fourier curve fitting, the details of which can be found in [24]. The Fourier curve fitted classic surge solution is prescribed as the unsteady inlet boundary condition for the straight cylindrical duct (which is the computational domain for the present work) and the flow inside is solved by using the incompressible Navier – Stokes equation to simulate surge in the duct. For the present analysis COMSOL MULTIPHYSICS™ version 3.5a is used as the computational solver. This solver uses a generalised version of the Navier – Stokes equation to allow for variable viscosity.

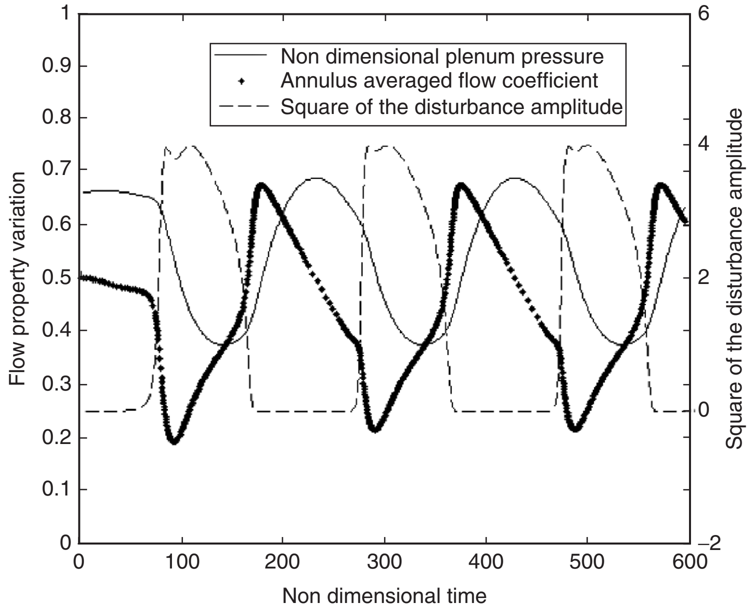

Figure 5. Non – dimensional variation of plenum pressure, annulus averaged flow and square of the disturbance amplitude of rotating stall with time during classic surge.

The incompressible Navier – Stokes solver solves for pressure and velocity vector components. Starting with the momentum balance in terms of stresses, the generalised equations in terms of transport properties and velocity gradients are

$$\rho \frac{\partial u}{\partial t} - V \cdot \left[\eta \left(\nabla u + (\nabla u)^T \right) \right] + \rho (u \cdot \nabla) u + \nabla p = F_v \tag{11}$$

$$\nabla \bullet u = 0 \tag{12}$$

Equation (11) is the vector form of momentum transport equation and eq. (12) is the equation of continuity for incompressible fluids. The variables and parameters appearing in the equations are the dynamic viscosity (η), density (ρ), velocity (u), pressure (p) and volume force field (F_v) such as gravity.

To study the nature of surge oscillations a straight cylindrical duct with a radius of 0.3 m having length to radius ratio of 4 is used. When surge occurs there will be oscillations in both the mass flow as well as the plenum pressure. The pulsating mass flow solution from the Moore – Greitzer model [1] is prescribed as the unsteady inlet boundary condition. This is because, as seen from Fig. 3, the duct inlet is in contact with the Moore – Greitzer compressor duct exit and hence the mass flow condition from the compressor exit will serve as the inlet boundary condition to the present computational domain. Thus the Moore – Greitzer solution for the axial flow coefficient is multiplied by mean blade speed to convert it to the dimensional form. The value of the mean blade speed used throughout the calculation is 14 m/s, which is a suitable value for a low speed compressor. On the curved walls, the no slip condition is specified. At the exit of the domain it is not possible to specify pressure since pressure oscillations due to surge is a part of the solution which is still not known. Therefore, the pressure has to evolve as a solution and this in turn will represent the plenum pressure variation with time since, as seen from our model compression system in Fig. 3, the computational domain opens to the plenum. Also the plenum pressure variations are assumed to be uniform throughout the plenum domain as stated earlier. Hence to begin with, the exit boundary is set to an initial value of pressure which is the same initial value of plenum pressure that has been used to solve the Moore – Greitzer equations. The reason for prescribing this condition is that the surge gets initiated at this value of plenum pressure. The plenum pressure coefficient is converted to its dimensional form (by using the standard values of $p_T = 101325\ Pa$, $\rho = 1.225$ kg/m^3 for air) and then fed to the computational solver as follows.

$$p_S = p_T + \Psi(\rho U^2) \tag{13}$$

In view of these, a neutral boundary condition is specified at the exit of the computational domain since this condition doesn't impose any constraint on the pressure. Hence the unsteady pressure variations at the duct exit also evolve as part of the solution. The neutral boundary condition implies an open boundary, no viscous stress condition. An open boundary refers to the duct exit opening to a large downstream volume (plenum in the present study), which is appropriate as per our model.

COMSOL MULTIPHYSICS™ version 3.5a uses the Finite element method to discretize the Navier – Stokes equations. The mesh generated for the computation employs tetrahedral elements inside the domain and triangular elements at the boundary. To determine the appropriate grid size, classic surge is simulated for different meshes by doubling the number of nodes and a grid independence test has been performed. Finally it has been found that a grid consisting of 31,973 nodes with 6,724 elements is accurate enough to explain the property variations. Since the surge oscillation frequency is low of the order of 10 Hz, a time step value of about 0.02 is used for all the computations as this value is enough to capture the oscillations.

The following convergence criterion is used for the computations. If U is the current approximation of the true solution vector and E, the estimated error in this vector, the software stops iterating when the relative tolerance exceeds the relative error that is computed as the weighted Euclidean norm

$$err = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{|E_i|}{W_i}\right)^2} \tag{14}$$

Here N is the number of degrees of freedom and $W_i = max(|U_i|, S_i)$ where S_i is a scale factor which is the average of $|U_i|$ for all DOF's i, times a factor equal to 10^{-5}. For all our present computations relative tolerance has been set to 10^{-6}.

3. RESULTS AND DISCUSSION

First the viscous surge flow behaviour is simulated in the cylindrical flow passage between the compressor duct and the plenum. Then control strategies are employed for controlling surge flow in this domain and the effects of surge control on plenum pressure oscillations are examined.

3.1. Simulation of the Viscous Surge Flow

While attempting to solve, the computational solver failed to capture the portion of the curve between the non – dimensional time ranges of 70 to 90, with respect to Fig. 5. This is possibly due to very abrupt drop in the axial velocity over a short interval of time, which the solver couldn't resolve. Another point to be noted is that the oscillating nature after the non – dimensional time of 100 follows a periodic variation for rest of the time. This prompted the authors to take an attempt to simulate the portion of the curve after this time range and it has been found that the solver could resolve the same. Therefore, for the results presented in this section, the starting point of the classic surge cycle is altered and the inlet boundary condition used is not the one corresponding to the peak point in the cubic axisymmetric characteristic. The plenum pressure value used to initialise the exit boundary of the domain is chosen corresponding to the altered initial value of the axial flow coefficient so that no discrepancy is introduced in the simulations. Thus the Moore – Greitzer solution used to simulate the classic surge is the same as that represented by the axial flow variation in Fig. 5; the only difference being that the simulation starts from a point later on the cycle. So in a real situation this is like flow simulation after the initiation of surge.

The surge occurs if there is an adverse pressure gradient in the fluid flowing system. Fig. 6 shows a plot of the variations in pressure difference between the inlet and exit of the duct with respect to time. The plot clearly indicates pressure build up at the duct exit periodically, which leads to surge. Also one could notice small pulsations in the regions just before the pressure reversal. This essentially indicates the presence of rotating stall like disturbances in the compressor flow that will act as surge precursors. This in fact is an outcome of the presence of periodic disturbance in the mass flow solution (represented by the square of the disturbance amplitude) of the Moore – Greitzer model as shown in Fig. 5.

It is shown in [24] that the present hybrid surge model is more realistic in predicting the surge oscillations than the Moore – Greitzer model [1] as it could explain the two important experimental observations – the presence of rotating stall disturbance that is triggered in a classic surge cycle, and the

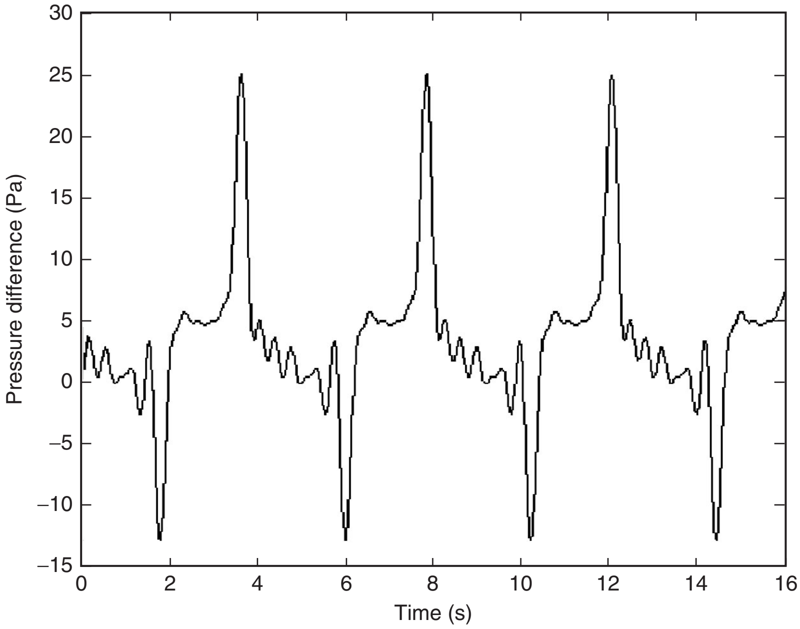

Figure 6. Pressure gradient during classic surge.

plenum flow being subjected to the same mean pressure with a superimposed high degree of fluctuation.

3.2. Feedback Technique to Control Surge

The focus of the surge control is to recover from this adverse pressure gradient. As discussed earlier the surge oscillation frequency is low of the order of 10 Hz. Hence in this paper results are presented for a surge control analysis in which steady feedback loops of suction and insertion are applied to the surge flow domain (cylindrical duct). This is achieved by simultaneous suction of mass flow close to the duct exit and insertion of the same quantity of flow close to the duct inlet. The surge occurs because of the periodic existence of adverse pressure gradient in the flow. By bleeding a proper quantity of the mass flow from the high pressure region close to the duct exit, which is responsible for the occurrence of surge, and reintroducing it close to the duct inlet it is perceived that recovery from the adverse pressure gradient could be attained. For doing this, holes are made on the curved cylindrical surface. Thus the results for a four hole and six hole surge control mechanisms are discussed in the present paper as follows.

In the four hole surge control mechanism two holes are employed, for suction of fluid from the main stream, at some specific locations on the duct and two other holes are used for insertion of the same quantity of fluid back to the main stream at some different locations. For the present case the centre of the insertion holes are provided on the curved cylindrical surface at a distance of 0.1 m from the inlet face of the cylinder and the suction holes are also provided at the same distance with respect to the exit face of the cylinder. Each of the two suction and insertion holes is located diametrically opposite with a diameter of 0.12 m. The computational mesh used for this study consists of 13,898 elements and 67,062 nodes since a denser mesh is required close to the holes. A steady flow velocity of 4 m/s has been employed through the control holes which correspond to a mass flow rate of 0.1112 kg/sec for the suction and insertion. When this case was tested 20% recovery from the adverse pressure gradient could be achieved. As seen in Fig. 7 the adverse peak pressure gradient (represented as the pressure difference between the inlet and exit of the duct) with four hole control mechanism is less compared to the value before control.

It is speculated that better flow mixing and hence more control can be achieved by employing more number of holes for suction and control. In order to establish this a six hole mechanism is also studied. In six hole surge control mechanism three holes are utilised for flow suction and three other holes are used for flow insertion purposes. The computational mesh used in this case consists of 13,254 elements and 63,688 nodes. In this mechanism, the centre of the insertion holes are located on the curved cylindrical surface at a distance of 0.15 m from the inlet face of the cylinder and the suction holes are provided at the same distance located with respect to the exit face of cylinder. Unlike for the four hole

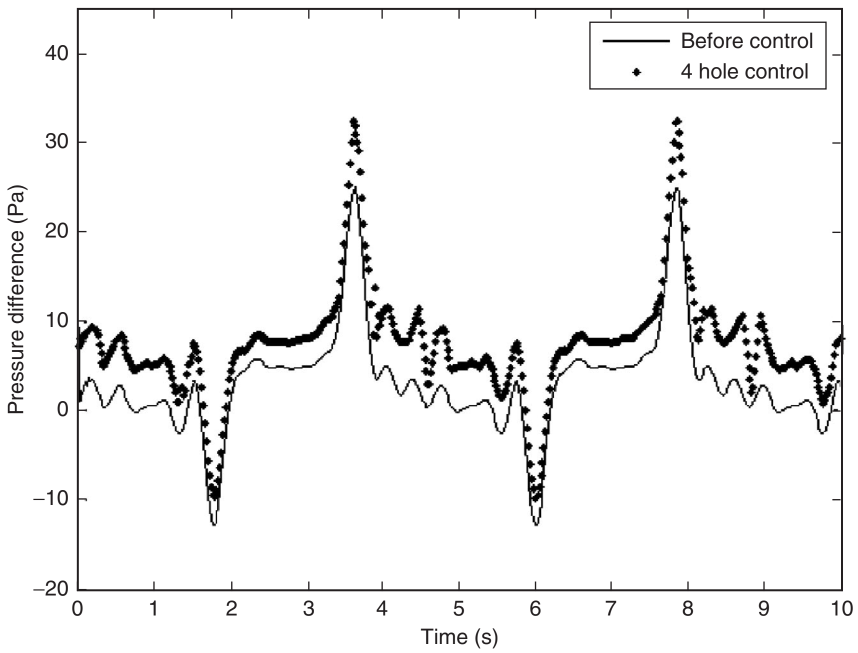

Figure 7. Temporal variation of pressure difference with four hole surge control mechanism.

surge control mechanism where the holes are provided at a distance of 0.1 m from the end faces of the duct, here the holes are provided at a distance of 0.15 m from the end faces of the duct. It is because of this reason that the mesh density is slightly low for the six hole mechanism. The two suction and insertion holes are located diametrically opposite to each other with the third pair of suction and insertion holes coming in between these two. All the suction and insertion holes have a diameter of 0.1 m. This case is put to test by first employing a steady flow velocity of 4 m/s through the control holes, which corresponds to a mass flow rate of 0.1159 kg/sec for the suction and insertion. Accordingly 52% recovery from the adverse pressure gradient has been achieved. Thus the effect of better flow mixing due to more number of holes is reflected in the result. When the steady value of flow velocity is increased to 7 m/s, which corresponds to a mass flow rate of 0.2028 kg/sec for the suction and insertion, complete recovery from the adverse pressure gradient in the duct has been obtained.

Figure 8 presents the difference between the duct inlet and exit pressure variation with time and thus

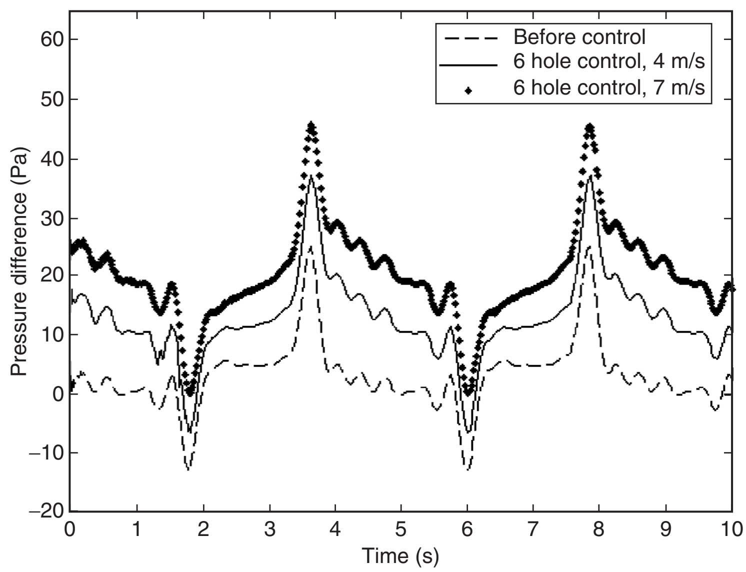

Figure 8. Temporal variation of pressure difference with six hole surge control mechanism.

the adverse pressure gradient recovery for the two control cases. It is clearly seen that 100% recovery is obtained with the steady control velocity of 7 m/s.

Figure 9 shows the temporal variation of the duct exit pressure before and after control. In the presence of control the rotating stall disturbance is visibly decayed. The high frequency fluctuations seen in the unsteady variation of plenum pressure before control are almost eliminated in the presence of a steady feedback control. When a steady feedback control of 7 m/s is employed, the temporal variation of plenum pressure has taken a periodic variation with an almost triangular waveform. This variation is smoother compared to the kind of fluctuations that existed without the surge control. Thus even if the plenum pressure is varying periodically with time, in the presence of a steady feedback control the variations are relatively more stabilised than without the feedback control. Since now the frequency of pressure oscillations have decreased the plenum pressure delivery is also expected to improve, thereby working in a more efficient manner. On the other hand the associated disadvantage is that there is a decrease in the value of pressure delivered by the plenum. This is expected because when feedback control technique is employed there is a decrease in the net mass flow of air entering the combustor thereby reducing the pressure level.

The compressor performance curve is plotted as the variation of the plenum pressure rise coefficient with the axial flow coefficient. The plenum pressure rise coefficient is the difference between the plenum and the ambient pressure, which is non dimensionalised by the quantity ρU^2 (see eqn. (2)). The axial flow coefficient is the ratio of the axial flow through the compressor to the mean blade speed (see eqn. (3)). The motion of the operating point in a compressor performance diagram in the presence of surge is termed as the limit cycle oscillation. Figure 10 shows limit cycle oscillation before control and in the presence of steady feedback control, which provided 100% adverse pressure gradient recovery. Although full recovery could be achieved, plenum pressure is still oscillating and the plenum pressure rise is reduced as seen earlier. However when the steady feedback control is employed the hysteresis that existed, during the surge oscillation without control, is almost eliminated. Thus, the energy loss from the flow is greatly reduced. This in turn will improve the work output from the downstream turbine, which leads to a greater cycle efficiency.

Figures 11(a) and 11(b) show the cross – sectional slice plot of velocity field with the 7 m/s control jet solved by the solver at two different times. Also shown is the cross – sectional slice plots of pressure field at different times in Figs. 12(a) to 12(c) for the same case. In order to check whether the feedback controller (which gave full adverse pressure gradient recovery) is practically feasible or not, the mass flow and the momentum ratio of control jets with respect to the main flow stream are determined. During computations employing different values of control mass flow rate a critical value of momentum ratio have been found, which is defined as the ratio of momentum of a control jet to that of the main flow

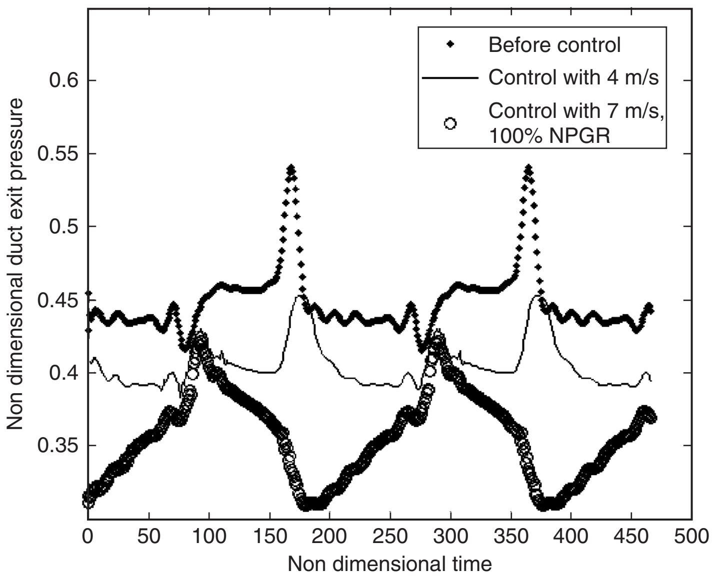

Figure 9. Non – dimensional variation of duct exit pressure with time before control and after using six hole feedback control mechanism.

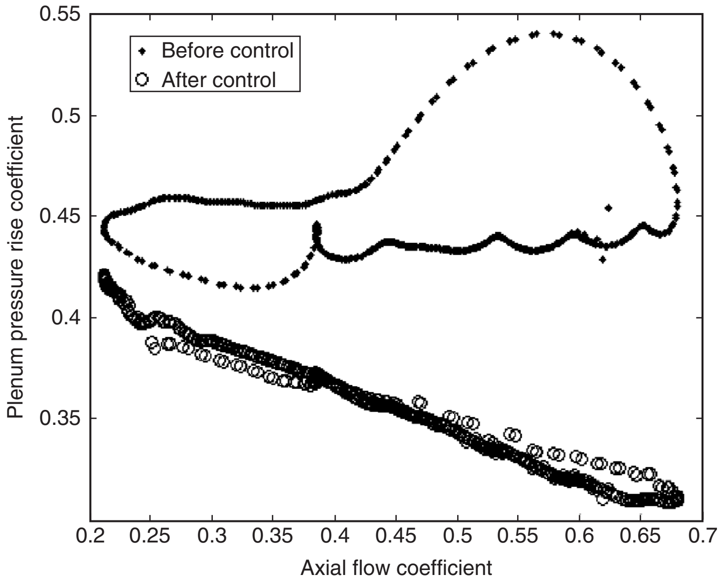

Figure 10. Limit cycle oscillation before and after the surge control with six hole feedback mechanism.

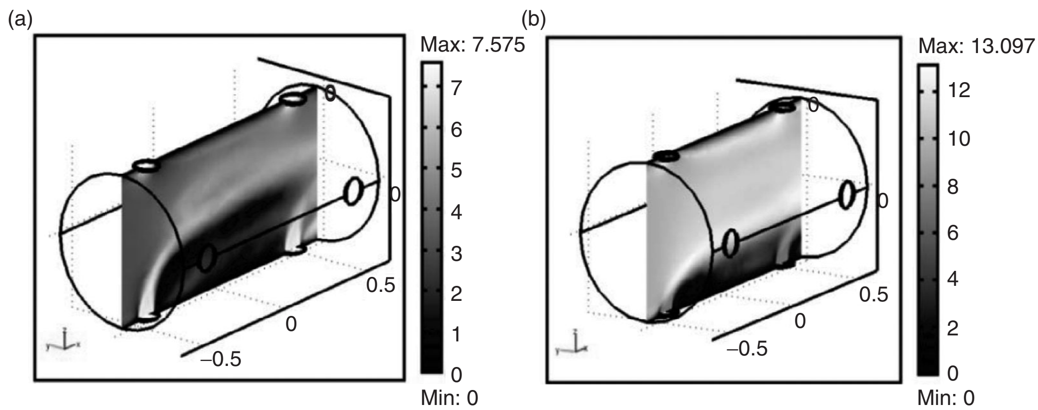

Figure 11. Velocity field (in m/s) with surge control at different times as calculated by the solver: (a) $t = 2$ sec and (b) $t = 8$ sec.

below which the computational solver fails to solve the model [24]. It has been found that for the case that is presented here the minimum momentum ratio per control jet (1.5%) is well above the estimated critical value (0.27%). Details of the calculation can be found in the appendix and in [24]. Thus it is ensured that the control jets will have well enough penetrating force to mix up with the main flow stream.

A plot indicating the variation, in percentage, of the bleed mass flow used for the feedback control with the adverse/negative pressure gradient recovery (%NPGR) is shown in Fig. 13. In this plot points are shown for the recovery achieved with a steady feedback control velocity of 4 m/s using the four hole surge control mechanism and the recovery provided by employing a control velocity of 4 m/s and 7 m/s with a six hole surge control mechanism. It is evident that increasing the number of control holes does provide a greater recovery from the adverse pressure gradient due to the effect of better flow mixing, as discussed earlier.

Thus in the present study primary emphasis is provided for controlling the viscous surge flow rather than controlling the inviscid one, as is usually done. After establishing that the proposed hybrid model is in agreement with some of the experimental observations, possibilities of employing a feedback control strategy for controlling the low frequency surge oscillations were investigated. The results seem to indicate that the feedback control is an effective technique to control the low frequency oscillations

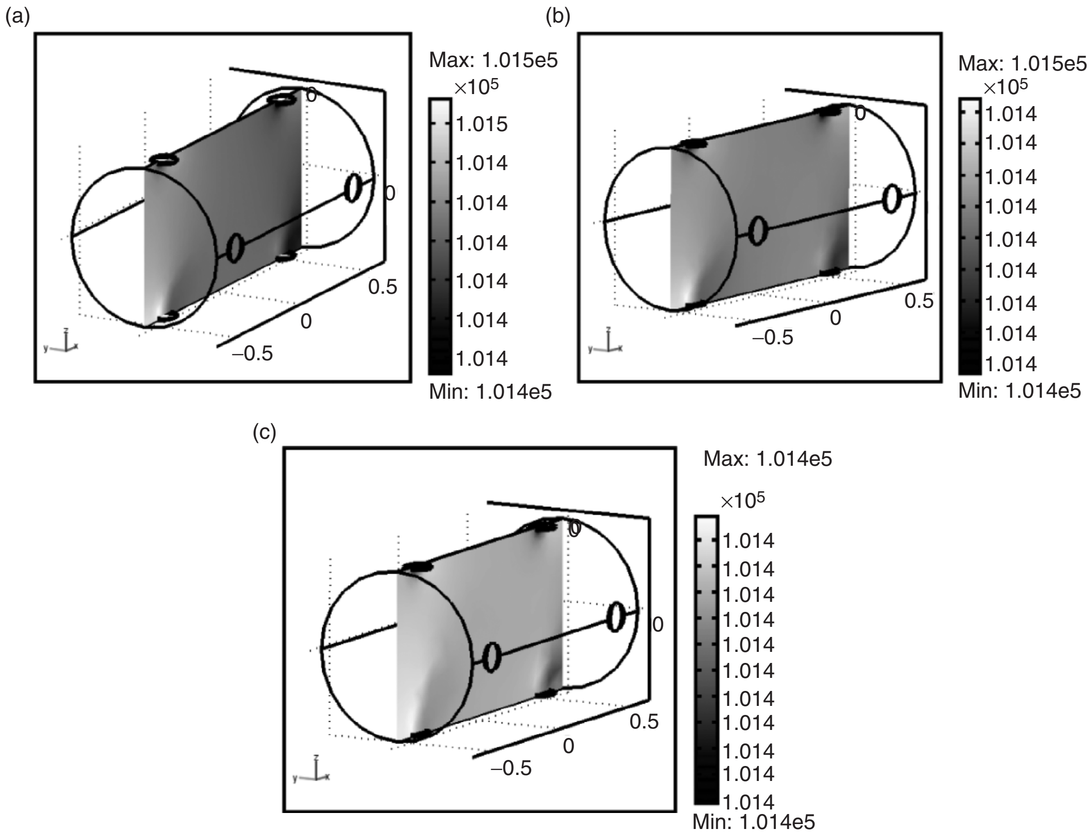

Figure 12. Pressure field (in Pa) with surge control at different times as calculated by the solver: (a) t = 3.6 sec, (b) t = 5.82 sec and (c) t = 6.2 sec.

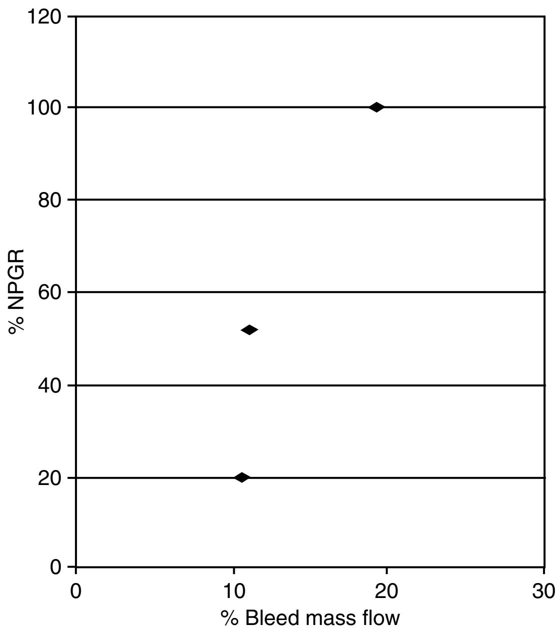

Figure 13. Negative pressure gradient recovery for different control mass flow rates.

as has been reported in the literature. It is also observed through the momentum ratio calculations (see appendix) that the proposed control strategy can be practically feasible.

In the present investigation, studies have been carried out only for steady values of feedback control. The behaviour of surge oscillations when time dependent controller is employed has to be investigated. It seems that the oscillations in the plenum pressure can be damped out by using a time dependent controller where time varying mass flow is employed for suction/insertion. The phase at which the bleed mass flow is removed and injected back for controlling is very important as it should be out of phase with the surge oscillation for the controller to be effective. The optimum location and dimension of control holes that need to be provided is also an important aspect that has to be investigated. The suction and injection of flow at an angle is another case that has to be looked upon. It is to be mentioned that in the present model the effects of compressibility and turbulence on flow control are not considered. It is perceived that this has to be investigated as a separate research topic.

4. CONCLUSIONS

(1) A hybrid approach is developed by coupling the Moore – Greitzer potential flow model and the viscous Navier – Stokes solution to simulate the axial compressor surge and a simple control strategy of feedback technique is proposed.

(2) Feedback control technique is adopted because of the low frequency nature of surge oscillations. Four hole and six hole steady surge control mechanisms are studied.

(3) The high frequency fluctuations seen in the unsteady variation of plenum pressure before control are almost eliminated in the presence of a steady feedback control. As a result the frequency of plenum pressure variations has decreased significantly and the pressure variations have now become smoother and are relatively more stabilised.

(4) The present study shows that complete recovery from the adverse pressure gradient is possible, but at the expense of a reduced pressure delivery by the combustor, by employing the six hole surge control mechanism. Increasing the number of control holes is shown to have better flow mixing and hence a greater surge control.

(5) When steady feedback control is employed both the hysteresis and the energy loss from the flow are greatly reduced. The reduction in energy loss is expected to improve the work output from the turbine, which can lead to greater cycle efficiency.

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NOMENCLATURE

A_c = flow through area of the compressor
a_s = sound speed
a = reciprocal time lag parameter of blade passage
B = non dimensional Greitzer parameter
C_x = compressor axial velocity
FEM = Navier – Stokes solution from the computational solver
H = semi–height of cubic axisymmetric characteristic in Fig. 2
J = square of the disturbance amplitude of rotating stall
K_T = constant throttle coefficient representing throttle curves in the compressor characteristic
L_c = effective length of the compressor duct
l_c = effective flow passage length through the compressor and its ducts
MG = Moore – Greitzer/Moore – Greitzer solution
m = compressor duct flow parameter
NS = Navier – Stokes solution
p_S = plenum pressure
p_T = ambient pressure
R = mean compressor radius
t = time
U = mean compressor blade speed
Vp = plenum volume
W = semi–width of cubic axisymmetric characteristic in Fig. 2

ψ_{c0} = shut off value of axisymmetric characteristic in Fig. 2
ϕ = local axial flow coefficient
Ψ = plenum pressure rise coefficient
ρ = density
Φ = annulus averaged axial flow coefficient
ε = non–dimensional time

APPENDIX

Mass flow ratio and momentum ratio calculations to check the feasibility of the proposed control technique:

In order to check whether the controller (which gave full adverse pressure gradient recovery) works or not it is essential to determine the mass flow and momentum ratio of the control jets with respect to the main flow stream.

Mass flow ratio:

With reference to Fig. 5,

Minimum velocity in the inlet velocity profile = $(0.21 * 14)$ m/s
= 3 m/s

Minimum mass flow entry rate through the duct inlet (corresponding to the minimum velocity point)
= $(\pi * 0.3 * 0.3) * 3 * 1.23 = 1.0433$ kg/s

Jet insertion velocity for the six hole flow control mechanism which provided full adverse pressure gradient recovery = 7 m/s

Total mass flow rate employed for injection = number of insertion control jets* mass flow rate through each hole
= $3*((\pi * 0.05 * 0.05) * 7 * 1.23)$kg/s
= 0.2028 kg/s

Maximum mass flow ratio = $0.2028 / 1.0433$
= 19.43%

Now to calculate the minimum mass flow ratio, repeating the same calculations shown above but by using the maximum velocity value in the inlet velocity profile, which is $0.67 * 14$ m/s = 9.5 m/s we find

Maximum mass flow entry through the duct inlet (corresponding to the maximum velocity point)
= $(\pi * 0.3 * 0.3) * 9.5 * 1.23$ kg/s
= 3.3038 kg/s

Minimum mass flow ratio = $0.2028 / 3.3038$
= 6.13%

Now to calculate the critical mass flow rate, it has been observed in the simulations that the solver failed to give a solution for injection velocities below 3 m/s. This is supposed to be the lack of penetrating force possessed by the control jet. Thus the injection velocity of 3 m/s is taken as the critical value.

Critical mass flow = $3*((\pi * 0.05 * 0.05) * 3 * 1.23)$ kg/s
= 0.0869 kg/s

Critical mass flow ratio = $0.0869 / 1.0433$
= 8.32%

Momentum ratio:

Maximum fluid force through the duct inlet = $3.3038 * 9.5$ N
= 31.386 N

Force of injecting fluid through one hole = $((\pi * 0.05 * 0.05) * 7 * 1.23) * 7$ N
= 0.4733 N

Minimum momentum ratio per hole = $0.4733 / 31.386$
= 1.5%

Now to find the critical momentum ratio,

Critical momentum of injecting fluid through one hole
= $((\pi * 0.05 * 0.05) * 3 * 1.23) * 3$ N
= 0.0869 N

Critical momentum ratio = $0.0869 / 31.386$
= 0.27%

Since the minimum momentum ratio is well above the critical value, the controller will work. Thus the proposed control mechanism is feasible to implement.

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