

ECONOMIES OF SCALE CAN BE IMPLEMENTED BY COOPERATIVE GAME: THE CHAIN MODEL CASE

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ABSTRACT

An important issue examined in this article is that the chain model (which solves for the optimal siting problem of regional wastewater treatment plants) cannot be implemented if the dischargers do not have fair cost sharing. The article finds that the total treatment cost of the optimal regional treatment plant is fairly assigned to each discharger by the N -cooperative game-MCRS approach. Therefore, the economies of scale are implemented by cooperative game in the chain model case.

INTRODUCTION

Arrangements of several sets of pipes can provide for efficient water quality control in certain situations because of economies of scale. Such a framework is efficient and provides significant savings but may not work in the real world. This is so because there may not be a regional authority that could help facilitate its evolution. In this case, a bargaining process may provide the necessary impetus to motivate prospective groups to bring about the necessary conditions for efficient regional water quality control. The appropriate bargaining process should be concerned with both efficiency and equity issues.

Zhu and ReVelle [1] use a siting model to solve the water quality control problem for regional wastewater treatment systems when the wastewater sources and treatment plants are arranged in a chain or linear situation (the chain model).

They found the efficient way to treat wastewater, but they did not address the problem of cost allocation among the dischargers. If any of the dischargers desiring to have a connection to the regional treatment plant are unwilling to pay the appropriate treatment cost, they could choose to build their own treatment plants. This could result in a big loss in efficiency.

The purpose of this article is to show how the above problem can be solved using an n -cooperative game theoretic approach. The n -cooperative game formulation used in this research is the minimum-cost, remaining-savings (MCRS) approach. The data in this article are adapted from Zhu and ReVelle [1]. The article finds that the total treatment cost of the optimal regional treatment plant is fairly assigned to each discharger by the N -cooperative game-MCRS approach. The discharger will pay the fair cost and decide to build the optimal treatment plant(s). Therefore, economies of scale are implemented as a cooperative game in the chain model case.

LITERATURE REVIEW

Heaney [2] used game theory to examine efficiency and equity aspects of environmental problems. The cooperative game is used as the theoretical basis for the allocation methods based on separable and non-separable costs. In the cooperative game, the separable costs are defined as the difference in cost between the player who joins the coalition and the player who does not join the coalition [3]. If each player is assigned its own separable cost, then those remaining costs that are not separable costs are called the “non-separable costs.”

Some cost allocation methods are based on separable and non-separable costs. They are the egalitarian non-separable cost (ENSC) method, the separable costs remaining benefits (SCRB) method, the minimum costs remaining savings method (MCRS), and the non-separable cost gap (NSCG) method.

The separable costs remaining benefits (SCRB) method was used in multipurpose water reservoir projects in the United States [4]. The method was widely used in multipurpose water development projects in the 1980s. The disadvantage of the SCR method is that it only analyzes coalitions of size 1, $N - 1$, and N . All other information is ignored. The bounds, upper and lower, for the core are determined by use of simple formulas.

The minimum costs remaining savings method (MCRS) is proposed by Heaney and Dickinson [5] and is a generalization of SCR. The bounds for the core in the MCRS method are obtained by solving a series of linear programming models and are as sharp as possible [6]. The egalitarian non-separable cost (ENSC) method is a naive method. That is because ENSC simply equally assigns the non-separable cost, which should be “proportioned” equally. The non-separable cost gap method (NSCG) is derived by the τ value, which is introduced by Tijds [7]. The method gets the bounds of the core by simple formulas and argues that the allocation of the non-separable cost is not only based on the remaining

alternate costs of the one-person coalitions, but also needs to be based on the remaining alternate costs of other coalitions.

If we compare these four methods by bounds for the core, the MCRS method is the best. That is because the method's bounds for the core are as sharp as possible. If we compare these four methods by subclass (convex and one-convex) cost games, MCRS shares with other methods their positive characteristics and becomes one of the best methods. Therefore, if MCRS method's computation cost is not too high, we should choose the MCRS method [6].

MODEL AND PROCEDURE

Faulhaber [8, p. 966] argues that one reason to let a member join a coalition is that the joint cost is less than the cost of acting independently. If the joint cost is higher than the cost of acting independently, they will choose not to join the coalition and act alone. Cooperative game theory can be applied systematically to make cooperative decisions [9, p. 1387]. Therefore, fairness must exist between the project members. Cooperative game theory operates as an N -person game and leads to three possible outcomes: 1) act independently; 2) join the grand coalition of all N player; or 3) form a coalition with only a subset (s) of the N players [9, p. 1387; 10, p. 52].

There are three general axioms that a fair solution to a cost should satisfy [5, p. 478]. The first axiom states that the costs assigned to the i th group, $x(i)$, must be no greater than their costs when they act independently, i.e.,

$$x(i) \leq c(i) \quad \forall i \in N.$$

The second axiom states that the total cost $c(N)$ must be apportioned among the n groups, i.e.,

$$\sum_{i \in N} x(i) = c(N)$$

If the two equations above are satisfied, the solutions are called "imputations." The third criteria is an extension of the first, requiring that the cost to each member must be less than or equal to the costs they would receive in any coalition S contained in N , i.e.,

$$\sum_{i \in S} x(i) \leq c(s) \quad \forall s \in N$$

All solutions satisfying the above criteria constitute the core of the game. Heaney and Dickinson [5, p. 478] argue that:

For sub-additive games the set of imputations is nonempty, but the core may be empty. A cost game has a convex core if

$$c(s) + c(T) \geq c(S \cup T) + c(S \cap T) \text{ for } S \cap T = \emptyset \quad S, T \subset N$$

In general, the more attractive (lower cost) the game is, the greater the chance that the core is convex. Conversely, the less attractive the game is, the greater the chance that the core is empty.

If the games have a core, we can find the upper and lower bounds on each $x(I)$ by

$$\text{Maximize or minimize } x(I) \quad (1a)$$

subject to:

$$x(i) \leq c(i) \quad \forall i \in N \quad (1b)$$

$$\sum_{i \in s} x(i) \leq c(s) \quad \forall s \in N \quad (1c)$$

$$\sum_{i \in N} x(i) = c(N) \quad (1d)$$

$$x(i) \leq 0 \quad \forall i \in N. \quad (1e)$$

If a game does not have a core, the solution to the linear programming problem will be infeasible. We then proceed by relaxing the values of the characteristic functions in the subgroup coalitions until we find a core. The solution can be found by solving the following linear program [11, p. 103]:

$$\text{Minimize } \theta \quad (2a)$$

subject to:

$$x(i) \leq c(i) \quad \forall i \in N \quad (2b)$$

$$\sum_{i \in s} x(i) - \theta c(s) \leq c(s) \quad \forall s \in N \quad (2c)$$

$$\sum_{i \in N} x(i) = c(N) \quad (2d)$$

$$x(i) > 0 \quad \forall i \in N. \quad (2e)$$

In summary, the procedure for the minimum-costs, remaining savings (MCRS) method is as follows [11, p. 103]:

- Step 1: Find the minimum [$x(I)$ min] and maximum [$x(I)$ max] costs that satisfy the core conditions graphically or by solving linear programs, equations 1 (core exists) or equations 2 (no core exists).
- Step 2: Prorate the non-separable cost (NSC) using:

$$\beta(i) = \frac{x(i) \text{ max} - x(i) \text{ min}}{\sum_{i \in N} [x(i) \text{ max} - x(i) \text{ min}]}$$

and

$$NSC = c(N) - \sum_{i \in N} x(i) \text{ min.} \quad (3)$$

- Step 3: Find the fair solution for each group using:

$$x(i) = x(i) \text{ min} + \beta(i) \text{ (NSC)} \quad (4)$$

DATA AND EMPIRICAL EXAMPLE

A numerical example is developed in this section based on one of the optimal and efficient regional plants in Zhu and ReVelle [1, pp. 141-142)]. In Zhu and ReVelle's article [1], the identification numbers, which are 3, 4, and 5, are in the same optimal regional treatment plant as used in this article. In Table 1, there are three waste sources (the dischargers 3, 4, and 5) and one optimal regional treatment (R), which was found by the siting model [1]. The data is the same as in Table 4 of their article and is only used for one of three optimal regional treatment plants. The reason for using only one of three optimal regional treatment plants is for ease in explaining the cost allocation.

Table 2 presents the cooperative treatment costs and the at-source treatment cost (no cooperation). For example, the at-source treatment cost of discharger 3 is \$2,916,336. The cooperative treatment cost of dischargers 3 and 4 is \$7,477,245. The costs presented in Table 2 will be used for computing the maximum and minimum costs for each coalition structure. The development of these costs is shown in the appendix.

In order to follow the three general axioms (see above) in a cost game to set the fair solution, the costs assigned to the dischargers 3, 4, and 5 need to meet the following conditions:

First, the costs assigned to the i th ($I = 3, 4, 5$) group, $X(I)$, must be no more than their costs when they acted independently. It is shown below.

Table 1. The Optimal Treatment Plant Location and Data

ID #	Flow (mgd)	Distance
3	3	15
4	4	5
5	29.4	2
R	0	

The data is the same as Table 4 in [1] and is modified. Since "14R," which is a potential treatment plant in their paper, is not an optimal treatment plant by their siting model result, "14R" is erased in the paper. Where R, which is "15R" in their paper, is the optimal regional treatment plant.

Table 2. Cost of the Various Coalition Sizes

Number of coalition	Discharger			Cost dollars
	3	4	5	
I	x			\$ 2,916,336
I		x		3,400,115
I			x	12,719,512
II	x	x		7,477,245
II	x		x	16,522,087
II		x	x	15,487,793
III	x	x	x	18,307,871

$$X(3) \leq 2,916,336$$

$$X(4) \leq 3,400,115$$

$$X(5) \leq 12,719,512$$

In the second, the total cost, $c(N)$, must be apportioned among the N groups, i.e.,

$$X(3) + X(4) + X(5) = 18,307,871$$

In the third, the criterion is extended from the first condition. That means that the cost to each group must be less than or equal to the costs they would receive in any coalition S contained in N . This is shown below.

$$X(3) + X(4) \leq 7,477,245$$

$$X(3) + X(5) \leq 16,522,087$$

$$X(4) + X(5) \leq 15,487,793$$

The upper bounds on $X(3)$, $X(4)$, and $X(5)$ have been set by the first three equations. The lower bounds on $X(3)$, $X(4)$, and $X(5)$ have been set by the last four equations. For example, in order to find the lower bound of $X(3)$, we can use the equations $X(4) + X(5) \leq 15,487,793$ and $X(3) + X(4) + X(5) = 18,307,871$. By subtracting 15,487,793 from 18,307,871, we can find that lower bound of $X(3)$ is 2,820,078. The following are all the lower and upper bounds situations.

$$2,820,078 \leq X(3) \leq 2,916,336$$

$$1,785,784 \leq X(4) \leq 3,400,115$$

$$10,830,626 \leq X(5) \leq 12,719,512$$

$$X(3) + X(4) + X(5) = 18,307,871$$

These bounds can also be solved by linear programming represented in equation (1) [or (2), if it is not in the core]. Table 3 presents these lower and upper bounds on costs for the three-discharger cost game.

In order to explain the MCRS method, we use the example for discharger 4 for the coalition (4, 5). Because of this cost game has a core, the upper and lower bound for discharger 4 can be generated by solving the following linear program shown in equation (1). The linear program for discharger 4 is as follows:

$$\text{Maximize or minimize } X(4)$$

subject to:

$$\begin{aligned} X(4) &\leq 3,400,115 \\ X(5) &\leq 12,719,512 \\ X(4) + X(5) &= 15,487,793 \end{aligned}$$

The result for maximum and minimum (lower and upper bound) treatment costs assigned to discharger 4 is shown in Table 3. These maximum and minimum values are essential to performing the MCRS solution procedure. The following are the steps for the minimum-costs, remaining savings (MCRS) method [11, p. 103] to find the cost allocation of discharger 4 and discharger 5:

- Step 1: Find the minimum [$x(4)$ min] and maximum [$x(4)$ max] costs that satisfy the core conditions graphically or by solving linear programs, equation (1) (core exists) or equation (2) (if no core exists).
- Step 2: Prorate the non-separable cost (NSC) using equation (3):

$$\begin{aligned} \beta(4) &= \frac{3,400,115 - 2,768,281}{(3,400,115 - 2,768,281) + (12,719,512 - 12,087,678)} \\ &= 0.5 \end{aligned}$$

Table 3. Lower and Upper Bounds on Costs for the Three Discharger Cost Game

Coalition	Discharger Bounds: L = Lower, U = Upper (\$)		
	Discharger 3	Discharger 4	Discharger 5
	L-U	L-U	L-U
(3), (4), (5)	None	2,768,281 – 3,400,115	12,087,678 – 12,719,512
(3, 4), (5)	*—	*—	None
(4), (3, 5)	*—	None	*—
(3, 4, 5)	2,820,078 – 2,916,336	1,785,784 – 3,400,115	10,830,626 – 12,719,512

*The coalitions in (3, 4), 5, and (4), (3, 5) are inessential, because they have less cost if they work individually.

and

$$\begin{aligned} NSC &= 15,487,793 - (2,768,281 + 12,087,678) \\ &= 631,834 \end{aligned}$$

- Step 3: Find the fair solution for each group using equation (4):

$$x(4) = 2,768,281 + 0.5 * 631,834 = 3,084,198$$

This procedure is also used for discharger 5 and the individual members of each coalition. Table 2 is used to compute the maximum and minimum costs for each coalition structure. Table 3 is found by using the linear programming problem presented in equation (1) [or (2), if the cost game is without a core]. The bounds (maximum and minimum cost) in Table 3 are used to calculate the MCRS solution for each coalition structure. Table 4 shows the total treatment cost for each coalition structure and the cost allocation for each discharger. Table 5 represents the cost saving for the various coalitions.

Table 4. Cost Allocation for Optimal Solution and Intermediate Solution

Coalition structure for least-cost solution	Total treatment cost	MCRS cost allocation (\$)		
		Discharger 3	Discharger 4	Discharger 5
(3), (4), (5)	19,035,963	2,916,336	3,400,115	12,719,512
(3), (4, 5)	18,404,129	2,916,336	3,084,198	12,403,595
(3, 4, 5)	18,307,871	2,871,191	3,058,641	12,378,038

Table 5. Cost Saving in Various Coalition Sizes

Size of largest coalition	Optimal coalitions	Total treatment cost	Percent saving
1	(3), (4), (5)	19,035,963	0.000
2	(3), (4, 5)	18,404,129	0.033
3	(3, 4, 5)	18,307,871	0.040

RESULTS AND DISCUSSION

The results for the coalitions, (4, 5) and (3, 4, 5), are consistent with the three general axioms (above) in a cost game to set the fair solution [5, p. 478]: 1) the treatment cost assigned to each discharger is not more than their treatment costs when they acted independently; 2) the total treatment cost is apportioned among the discharges; and 3) the treatment cost assigned to each member (dischargers) is less than or equal to the treatment cost that they would receive in the coalition, S , contained in grand coalition, N . These two coalitions are feasible coalitions and fall within the core of the cost game. Table 4 shows that no participant pays more than its cost of acting independently.

Cost savings shown in Table 5 are significant when we compare with the cost of acting independently. Because of both individual saving and coalition structure saving are shown in Tables 4 and 5, it presents the chain model that has an economy of scale situation.

One disadvantage of the chain model is that it ignores transaction costs. Heaney [11, p. 103] argues that: "As the size of the groups grow, transactions costs would be expected to increase at the margin due to multiple political jurisdictions, growing administrative costs, shifting environmental impacts, etc." The other disadvantage of the chain model is that the information for cost sharing is hard to get when there are too many dischargers. Therefore, if the chain model yields an optimal regional treatment plant that includes many dischargers, we might consider finding an alternative potential treatment plant that includes fewer dischargers.

CONCLUSION

In water quality control cases, analysts are looking for the most efficient technique. The efficient solution usually involves economies of scale, for example, the sitting location of the optimal regional wastewater treatment plant. The chain model finds the numbers and the location of the regional facilities at the least total cost (transfer cost and treatment cost). Unfortunately, the chain model ignores the cost allocation problem. In addition, if the dischargers do not pay a fair cost, they may decide to build their own treatment plants, and a big loss in efficiency will take place. The article finds that the total treatment cost of the optimal regional treatment plant is fairly assigned to each discharger by the N -cooperative game-MCRS approach. Therefore, the economies of scale are implemented by cooperative game in the chain model case.

In a future study, if there is a big group of dischargers in the same optimal regional treatment plant, the chain model (the optimal and the most efficient technique) may be changed to a "good" model (nearly optimal model), which has fewer dischargers than the optimal model (solved in chain model) has, because the transaction cost is getting higher and the information is hard to get.

APPENDIX

The procedures for developing the data shown in Table 2 in the text of the article are described in this Appendix. The information for example 2 (Delaware data) on pages 141-142 of [1] were used.

Consider coalition 1. The first three cost figures in Table 2 were derived using the treatment cost function for example 2 on page 141 of Zhu and ReVelle [1]. This cost function is as follows:

$$C_{1i} = 1,838,377 Q^{0.35364} \quad Q = 0.0 - 2.0 \text{ mgd} \quad (\text{A.1a})$$

$$C_{2i} = 1,622,888 Q^{0.53351} \quad Q = 2.0 - 5.0 \text{ mgd} \quad (\text{A.1b})$$

$$C_{3i} = 1,334,286 Q^{0.65517} \quad Q = 5.0 - 20.0 \text{ mgd} \quad (\text{A.1c})$$

$$C_{4i} = 980,502 Q^{0.75801} \quad Q = 20.0 - 100.0 \text{ mgd} \quad (\text{A.1d})$$

where I denotes the discharger.

The calculations for discharges 3, 4, and 5 (at source of treatment) are as follows:

$$C_{23} = 1,622,888 * 3^{0.53351} = 2,916,336 \quad (\text{A.2a})$$

$$C_{24} = 1,622,888 * 4^{0.53351} = 3,400,115 \quad (\text{A.2b})$$

$$C_{45} = 980,502 * 29.4^{0.75801} = 12,719,512 \quad (\text{A.2c})$$

The cost calculations for coalitions II and III assume that the dischargers transfer their waste to a regional treatment plant R . Hence each discharger's cost will include a transport cost as well as a (regular) treatment cost. The transport cost function shown for example 2 on page 141 of [1] along with the respective (regular) treatment cost function shown for example 2 on page 142 of [1] is used to derive the various cost figures shown in Table 2 for coalitions II and III. The transport cost function is written as:

$$T_i = 51,577 Q^{0.598}/\text{mi} \quad (\text{A.3})$$

where I denotes the discharger. The (regular) treatment cost function is stated as:

$$R_i = 349,514.25 Q + 2,382,405 \quad (\text{A.4})$$

where I denotes the discharger.

Now consider the cost of the coalition consisting of discharges 4 and 5 where their waste is treated in regional treatment plant R . The transport cost for the coalition is as follows:

$$T_3 + T_4 = (51,577 * 3^{0.598}) * 15 + (51,577 * 7^{0.598}) * 7 \quad (\text{A.4a})$$

$$T_3 + T_4 = 2,648,240. \quad (\text{A.4b})$$

The (regular) treatment cost for the coalition is as follows:

$$R_3 + R_4 = 349,514.25 (3 + 4) + 2,382,405 \quad (\text{A.5a})$$

$$R_3 + R_4 = 4,829,005. \quad (\text{A.5b})$$

The total coalition cost in this case is \$7,477,245. The cost calculations for the other coalitions shown in Table 2 are based on the same method.

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