J. ENVIRONMENTAL SYSTEMS, Vol. 27(2) 127-149, 1999

RISK-BASED DESIGN OF AIR STRIPPING TOWERS WITH FUZZY SET THEORY

JON W. SCHAUBLE Burns and McDonnell, San Diego, California

BRUCE I. DVORAK University of Nebraska, Lincoln

ABSTRACT

Frequently, environmental engineers are faced with uncertainty in making design decisions, because the true value of many process parameters is unknown. As an alternative to the probabilistic approach, fuzzy logic can be used for developing a risk-based design methodology. When fuzzy logic methods are compared to probabilistic methods, they yield similar results for the same design conditions. However, fuzzy logic requires fewer assumptions about the uncertainties and less computing power than probabilistic methods. In this case study, the design of air stripping towers was modeled, taking into account uncertainties in mass transfer and in the Henry's constant. For the design of air stripping towers, it was found that in addition to cost, the risk of failure is an important design consideration. When a wide range of designs is examined, it is clear that a significant overdesign is both cost-effective and more reliable.

INTRODUCTION

The purpose of the article is to present a risk-based design methodology using fuzzy sets for air stripping towers and compare it to the more common probabilistic approach. Environmental engineers must design treatment systems to meet remediation goals for a wide range of contaminants in water, air, and soil. However, environmental engineers are faced with uncertainty in making design decisions, because the true value of many process parameters is unknown. To avoid building a system that does not achieve the regulatory treatment goal,

127

© 1999, Baywood Publishing Co., Inc.

systems are over-designed (i.e., designed to achieve a treatment goal more stringent than the regulatory goal). For many processes, a range of designs can achieve the same treatment goal, but some of these designs will be more sensitive to uncertainty than others. Faced with this complexity, design engineers require a rational method for assessing and comparing the uncertainties inherent in each design.

Uncertainties are often mathematically modeled using a statistical approach; relevant parameters are assumed to be random variables with well-known probabilistic distributions. Monte Carlo simulation techniques can be used to determine the statistics of the output parameters. An example of the application of this approach is provided by Freiburger et al. [1], where uncertainties in a contaminant's Henry's Constant and mass transfer coefficients were examined. This probabilistic approach assumes that all parameter uncertainties are random, independent, and can be described by a known probability density function. However, these assumptions may not be true. Frequently uncertainty arises not from randomness but from incomplete data, vague descriptions, or subjective interpretations of expert judgments, where there is little validity to assigning a specific probability density function.

Fuzzy set theory is fundamentally different from probability theory. The theory of probability is concerned with the frequency of occurrence of a sample in a population. Fuzzy set theory deals with the closeness of a sample to resembling an ideal element of a population or the accuracy of the sample in representing the population [2]. The two theories have similarities, despite their fundamental differences. For example, both theories can be used to quantify the likelihood of an event.

Packed tower air stripping has been established as the most popular groundwater treatment method for removing volatile contaminants because of its low capital and operating costs. An air stripping tower is a tall cylinder filled with a porous packing material. Typically, the system operates in a countercurrent mode where the contaminated water is sprayed on top of the packing, while a blower forces air up through the packing from the bottom. As the water and air pass through the tower, the volatile contaminants transfer from the liquid phase into the gas phase. The contaminant-laden air is either discharged to the atmosphere or treated further, depending on the local air quality regulations.

The design removal efficiency represents the contaminant removal achieved if the mean values of Henry's constant and the mass transfer coefficients are used. Risk of failure in an air stripping tower design represents the likelihood, given the uncertainty in Henry's constant and the mass transfer coefficients, that the tower will meet the treatment goal. In this research, risk of failure of air stripping towers does not consider factors such as pump failure, fouling of the packing material, or electrical failure.

In this article, fuzzy set theory will be applied to consider treatment process uncertainty in a risk-based approach. A study of the design of air stripping towers is used to compare fuzzy and probabilistic methods and to provide an example application for the determination of reliable design strategies using fuzzy logic.

Risk of failure is an important design consideration for air stripping, because it is more cost-effective to overdesign a tower than to retrofit an existing tower to increase its removal efficiency. The reliable design of an air stripping tower depends on accurate estimates of Henry's constant and the mass transfer coefficients. In this study, fuzzy set theory (specifically fuzzy numbers defined subsequently) is used to represent uncertainty in Henry's constant and the mass transfer coefficients.

DISCUSSION OF PREVIOUS RESEARCH

Fuzzy Set Theory

In general, fuzzy set theory is an extension of the classical binary logic. Fuzzy set theory is considered to be a theory of logic that interpolates the 0 (false) and 1 (true) truth values of "crisp" logic to the whole real interval (0, 1) [3]. The central concept of fuzzy set theory is the membership function, which numerically represents the degree to which a given element belongs to the set. Formally a fuzzy set is defined as X which is a universe set of elements. A is called a fuzzy (sub)set of X, if A is a set of ordered pairs:

$$A = \{ (x, \mu_A(x)), x \in X, \mu_A(x) \in [0, 1] \},$$
(1)

where $\mu_A(x)$ is the grade of the membership of x in A. The closer $\mu_A(x)$ is to 1, the more x belongs to the set A; the closer it is to 0, the less it belongs to A. In this way fuzzy sets allow flexibility in expressing uncertainties with regard to set definitions such as the set of possible values for the Henry's constant of Chloroform.

The application of fuzzy set theory has penetrated into many areas of environmental engineering [4-7]. One useful application of fuzzy logic is to use a special case of fuzzy sets, the so-called fuzzy numbers. Fuzzy numbers can be used to represent imprecise parameters in analytical or empirical models. Figure 1 shows an example of a fuzzy number. The alpha value is the membership level of a given parameter value. The larger the alpha (membership) value, the greater the creditability that this value accurately describes the parameter. The vertices of the triangular fuzzy number shown in Figure 1, can be defined constructed from an expert appraisal: "What is the smallest value given to the uncertain parameter? What is the highest? And if we were allowed to give only one value, what value is the most credible (likely) value?" The most credible value is given a membership value of 1; the numbers that fall short of the smallest possible value and those that exceed the highest possible value are assigned a membership value of 0. Intermediate membership grades between there vertices are obtained here by



Figure 1. Triangular fuzzy number.

linear interpolation. Additional fuzzy number shapes can be used, as discussed in the Methodology.

Dong and Shah's vertex method of performing mathematical operations on fuzzy numbers was used in this research [8]. The application of this method is an involved and extensive procedure. According to this method, if a function of *n* fuzzy numbers is continuous in the dimensional region, and no extreme points exist in this region, then the solution for any mathematical operation is the interval with endpoints being equal to the minimum and maximum of the function at the vertices of the region defined by the fuzzy numbers. The solution vertices are all of the combinations of the interval endpoints (maximum and minimum point) at each alpha value for the fuzzy numbers. Putting all of the solution intervals together on a graph with their respective alpha values gives the membership function of the solution. Figure 2 shows the simple addition of two fuzzy numbers A and B. The interval boundaries of the alpha values of the resulting fuzzy number C in this case are obtained by simply adding the minimum and maximum possible values at each alpha value.

Fuzzy numbers can be used to assess the risk of failure of designs. Ang and Tang proposed a "probabilistic reliability index" for quantifying the risk of failure of a system in engineering planning and design [9]. Borrowing from this

FUZZY SET THEORY / 131



Figure 2. Addition of two triangular fuzzy numbers.

work, Shrestha and Duckstein proposed a "fuzzy reliability index" for structural failure, which resembles the probabilistic reliability index of Ang and Tang [10]. The fuzzy solution to a structural system is calculated by subtracting the fuzzy number representing the load from the fuzzy number representing the strength.

$$Z = Strength - Load$$
(2)

The resulting fuzzy number (Z) is the risk measure or margin of safety. Failure occurs when the fuzzy load exceeds the fuzzy strength, or Z is less than zero. Success occurs when the fuzzy strength exceeds the fuzzy load, or Z is greater than zero. The fuzzy risk index is calculated in order to quantify the risk of failure of the system.

Shrestha and Duckstein applied their fuzzy risk index to structural loading; however, the concept can be applied to any system with a fuzzy solution and failure criteria [10]. In this research, the system with a fuzzy solution is the air stripping design equation, and the failure criteria is removal efficiency. Since fuzzy membership functions are not the same as probability density functions, there is a need for different nomenclature. Thus, the risk determined with fuzzy logic is typically quantified as a possibility of failure rather than a probability of failure.

Air Stripping Theory

Air stripping is a common treatment technology for water contaminated with volatile organic compounds. Air stripping involves the mass transfer of the contaminant from the liquid phase to the gas phase, often in a packed tower. Henry's Law describes the partitioning of the contaminant between its liquid and gas phase at equilibrium. Henry's constant is a measure of a chemical's volatility in water, and it is often considered dimensionless; but, actually it has dimensions of volume of liquid divided by volume of air. Often, only a limited number (< 10) of experimentally determined Henry's constant values exist, making it difficult to identify a probability density function with a true mean and variance.

An equation for designing countercurrent air stripping towers can be obtained from a mass balance over a control volume [11]; the design equation assumes steady-state conditions, dilute solutions, chemical equilibrium, and absence of the contaminant in the surrounding air. The design equation solved for the ratio of the influent and effluent contaminant concentrations is as follows:

$$\frac{C_{LL}}{C_{LE}} = \left\{ exp\left[\frac{h_T K_L a A}{Q_L} \left(\frac{S-1}{S}\right) - \frac{1}{S}\right] \right\} \left(\frac{S}{S-1}\right)$$
(3)

where A is the cross-sectional area of the tower, K_{La} is the overall mass transfer coefficient, and C_I and C_E refer to the influent and effluent contaminant concentrations, respectively. The stripping factor, S, is dimensionless and defined as follows:

$$S = \frac{Q_G H}{Q_L}$$
(4)

where Q_G and Q_L are the volumetric gas and liquid flow rates, respectively. The ratio Q_G/Q_L is often called the *air-to-water ratio* and is commonly used to characterize the amount of air blown through the tower.

The Onda correlations were used to predict the mass transfer coefficient (K_La) [12, 13]. The Onda correlations are a set of three empirically-based equations that estimate the liquid-phase mass transfer coefficient, gas-phase mass transfer coefficient, and specific interfacial area. These three parameters and the Henry's constant are used to calculate the overall mass transfer coefficient.

Analysis of Air Stripping Risk of Failure

This study considered the uncertainty of air stripping towers with the same contaminants and conditions as another study that used probabilistic methods. Freiburger et al. [1] used Monte Carlo simulations to assess the risk of failure of air stripping towers. In the Freiburger study, Henry's constant and the mass transfer coefficients were treated as random variables. The Monte Carlo

simulations assumed independence between the variation in Henry's constant and variation in the mass transfer coefficients.

Freiburger et al. [1] gathered Henry's constant data from a variety of sources; a cumulative density function, which assumed that each Henry's constant value had equal importance, was constructed from the data. Freiburger handled the mass transfer coefficients in a similar manner to this study: the Onda correlations were used to estimate the gas-phase and liquid-phase mass transfer coefficients, and these were multiplied by an error factor. This error factor was the same as that reported by Onda et al. [13]: \pm 30 percent for gas phase and \pm 20 percent for liquid phase. Freiburger assumed that these errors represented one standard deviation from the mean for a normally distributed population (68% confidence interval) and created normally distributed cumulative density functions for the gas and liquid-phase error factors. Actually, the correlation errors reported by Onda et al. [13] were associated with 90 percent confidence intervals. Also, these errors may not be normally distributed but instead biased in one direction [14-16].

The random variables in the Monte Carlo approach (mass transfer error factors and Henry's constant) were assumed to be independent. If the variables were in fact dependent, then the results of the Monte Carlo simulation are in question. In contrast, the fuzzy method implicitly considers possible dependencies between parameters, because fuzzy mathematics considers the possibility of simultaneous extreme values and accounts for any dependencies that may exist [17].

For each Monte Carlo simulation, a random number generator picked the values of Henry's constant and the mass transfer error factors from the cumulative distributions described previously. Then using these values and the same air stripping design equations that were used for this study, the actual removal efficiency of the tower design was calculated. If the design did not achieve 95 percent removal, the design was recorded as a failure. This process was repeated many times, and the number of failures divided by the total number of runs was equal to the probability of failure for that tower design. Freiburger performed a sensitivity analysis and found that 200,000 simulations were required to ensure accuracy of the results [1, 18]. In contrast, fuzzy set theory can be used to calculate the entire fuzzy distribution of a tower design with one simulation. Due to the enormous number of simulations that were required for the Monte Carlo approach, only a small range of designs were examined by Freiburger's study.

The conclusion reached from Freiburger's study was that, all things being equal, lower A/W ratios lead to lower probabilities of failure, and thus more reliable designs. Unfortunately, Freiburger could not explain why this conclusion was true, and stated the limitations of his study:

... It is certain that other designs exist which could achieve the same probability of meeting 95% removal at lower cost. However, the search for such

designs requires a more exhaustive design approach which entails evaluating a wider spectrum of possible designs (i.e., designs for which analysis would predict design removal rates other than 95%) [18].

Because of the simplicity of performing simulations using fuzzy set theory, a wider range of air stripping tower designs were examined with removal efficiencies other than the removal goal (95%) in this study. Freiburger's conclusion was found to hold true only under certain conditions, as described subsequently.

METHODOLOGY

To apply fuzzy set theory to air stripping, design parameters that contain uncertainty must be identified, and fuzzy numbers must be created for these parameters. The Henry's constant of a contaminant has considerable uncertainty, because it is sensitive to temperature variations and impurities in the water. Mass transfer coefficients are typically estimated using empirical correlations that contain a significant degree of error. Also, Henry's constant and the mass transfer coefficients are the only parameters in the air stripping design equation which the designing engineer has little or no control over. For these reasons, Henry's constant and the mass transfer coefficients were chosen to model as fuzzy numbers.

Many fuzzy numbers (shapes) can be created from the same data. The triangle is a common shape for fuzzy numbers, because it is simple to construct. A narrow or wide triangle could be constructed depending on the range of extreme values used. If the most extreme possible (outlier) values of a parameter are used, then the triangle is wider at its base, compared to a triangular fuzzy number constructed using the "extremely likely" values, which ignore values that are determined by an expert to be outliers of questionable validity.

For the trapezoidal fuzzy shape, the median (most likely) value is not used. Instead, two intervals are required for the creation of a trapezoidal fuzzy number. The two intervals can be rationalized as the "likely" maximum and minimum values and the "extreme" possible maximum and minimum values of the parameter in question. Another fuzzy shape examined was the "hat shape," which uses the two intervals used for the trapezoidal shape (likely and extreme), as well as the median value. The fuzzy shapes examined in this research are shown in Figure 3. These shapes were compared and contrasted within the context of air stripping, and the findings are discussed in the results.

Fuzzy numbers were developed for the three chemicals studied in this research: trichloroethylene (TCE), tetrachloroethylene (PCE), and bromoform (CHBr₃). These common water contaminants were selected, because they represent a wide range of volatilities. The fuzzy number for each contaminant was determined by collecting all the available data from the technical literature. Both experimentally determined and thermodynamically calculated values of Henry's constant were obtained from 10, 11, and 8 sources for TCE, PCE, and



FUZZY SET THEORY / 135

Figure 3. Fuzzy shapes examined.

bromoform, respectively. The Henry's constant data was used to create the fuzzy numbers for the Henry's constant of each chemical. The median value of the Henry's constant data was used as the "most likely" value and was assigned a membership level equal to 1.0. The spread of the Henry's constant data determined the width of the fuzzy number at a membership level of zero. The fuzzy numbers for the Henry's constant are listed in Table 1.

An overall coefficient (calculated from liquid-phase measurements), K_La , for mass transfer across the gas/liquid interface must be obtained to design an air stripping tower; it is a parameter in Equation 2. K_La is often represented as the sum of the gas-side and liquid-side coefficients (the two-resistance theory):

$$1/K_{L}a = 1/k_{L}a + 1/(H k_{G}a)$$
(5)

where k_{Ga} is the gas-side mass transfer coefficient, and k_{La} is the liquid-side mass transfer coefficient. Equation 5 is predicated on the assumption that equilibrium exists at the gas/liquid interface. Experimental evidence supporting the two resistance theory was presented by Sherwood and Holloway [19]. The gas- and liquid-side mass transfer coefficients (k_{La} and k_{Ga}) can be estimated by the Onda correlations.

The fuzzy numbers for the mass transfer coefficients (k_La and k_Ga) were not determined in the same manner as Henry's constant. Crisp (non-fuzzy) values of the liquid and gas-phase mass transfer coefficients were estimated using the Onda correlations [12, 13]. Then these crisp mass transfer coefficients were multiplied by fuzzy error factors in order to represent uncertainty in the Onda correlations. Onda reported \pm 20 percent error between predicted and observed values in the estimate of the liquid-phase mass transfer coefficient. The extreme minimum and maximum values for both the liquid-phase and gas-phase error factors were determined from the original correlation plots [12, 13]. The fuzzy numbers for the k_La and k_Ga multiplier factor are shown in Table 1.

	Henr	'y's Consta		Ko Error	
Description	Bromoform	TCE	PCE	Factor	Factor
Extreme minimum	0.0093	0.12	0.29	0.62	0.57
Likely minimum	0.0095	0.17	0.31	0.80	0.70
Median value	0.010	0.18	0.34	1.00	1.00
Likely maximum	0.013	0.24	0.40	1.20	1.30
Extreme maximum	0.016	0.29	0.45	1.58	2.14

Table 1. Data Used to Create Fuzzy Numbers for Henry's Constant (10⊃C) and the Mass Transfer Error Factors^a

^aHenry's constants have units of volume of liquid/volume of air (m³ liquid/m³ gas).

In order to analyze the range of possible air stripping tower designs, a matrix of simulations were created. Many parameters in the air stripping tower design equation were held constant. For all of the simulations, a temperature of 10°C, packing material of 7.6-cm (3-inch) plastic saddles, and a water flow rate of 5.68 m³/minute were held constant. A range of unique combinations of tower height, diameter, and A/W ratio can result in a tower design with the same removal efficiency for a given contaminant; a matrix of unique designs was created for each chemical and removal efficiency by varying the three parameters. Tower diameters varied from 1.55 to 6.94 meters, while the height of the towers varied from 4.82 to 22.8 meters. Air-to-water ratios (A/W) between 5.9 and 800 were examined.

The Fuzzy Air Stripping Analysis Program (FASAP) was developed in order to assess the risk of failure of each design based on the inherent uncertainty in Henry's constant and the mass transfer coefficients. The characteristics of each design were entered into FASAP, and the fuzzy removal efficiency was calculated using the vertex method. Equation 6 was used to determine the possibility (or risk) of design failure for each simulation result, such as that shown in Figure 4.

$$P(failure) = \frac{Failure Area}{Total Area}$$
(6)

In Equation 6, failure is defined as a design that does not achieve the required treatment goal. In this study, failure does not include the possibility of structural failure, equipment failure, or fouling of the packing material. The possibility (or



Figure 4. Example of fuzzy solution and area of design failure.

risk) of failure of the air stripping tower is equal to the failure area divided by the total area. The failure area is defined as the area under the fuzzy removal efficiency curve that is less than the removal goal, while the total area is the entire area under the fuzzy removal efficiency curve. The failure area depends on the treatment goal. In this study, three removal goals (92%, 95%, and 98% removal) were used for each tower design.

Figure 4 shows an example of a fuzzy solution from FASAP. The area under the curve below the required removal efficiency is the area of design failure. In this case, the design removal efficiency of the tower and the required removal efficiency are both equal to 95 percent. The design removal efficiency represents the contaminant removal achieved if the most likely (median) values of Henry's constant and the mass transfer coefficients are used. Clearly, the area of design failure is over half of the total area under the curve, leading to a possibility (risk) of failure over 50 percent. In order to prevent such unreliable designs, air stripping towers are typically overdesigned (i.e., designed to have a design removal efficiency significantly greater than the required removal efficiency). For this reason, nearly all of the designs examined in this research were overdesigned.

After evaluating the risk of failure for an air stripper, the cost of each design was estimated. A cost model by Dvorak et al. was used in this study [20]. Dvorak's model is a modification of the Adams and Clark model [21]. The Dvorak model incorporates direct capital cost, operational cost, and maintenance cost. The present worth of the project was calculated for the given design life and then divided by the volume of treated water to obtain a unit treatment cost. Table 2 lists the economic assumptions that were used in this study.

RESULTS

Fuzzy Shape Comparison

Four shapes were examined for use as fuzzy numbers: wide triangle, narrow triangle, trapezoid, and hat shape. In order to compare the results from the

Parameter	Value			
Electric cost	\$0.057/kW-hr			
Labor cost	\$16.50/hr			
Capital-recovery period	5 and 25 years			
Capital-recovery interest rate	10%			
ENR construction cost index	4652 (April 1990)			
Net efficiency of blower motor	35%			
Net efficiency of pump motor	80%			

Table 2. General Economic Assumptions

different fuzzy shapes, the same data was used to create all of the fuzzy shapes for the Henry's constant of each contaminant and the mass transfer error factors. Descriptive terms such as extreme minimum, likely minimum, median, likely maximum, and extreme maximum were used to create the fuzzy numbers as described in the methodology. The values for each term are listed in Table 1.

A comparison was made between the results obtained by using the different fuzzy shapes for two contaminants, each with unique Henry's constant data. Figure 5 illustrates a range of designs that achieve the TCE removal goal (95%); the tower height varied, but the stripping factor (4.0), diameter (1.75 m), and flowrate (5.68 m³/min.) were held constant. Thus, the mass transfer rate (K_La) was the same for all designs shown in Figure 5. The TCE removal goal for all of the designs was 95 percent, however, the design removal efficiencies varied from 91 percent to 99 percent. In this study, the risk of failure represents the likelihood, given the uncertainty in Henry's constant and the mass transfer coefficients, that the tower will not meet the treatment goal of 95 percent removal.

As shown in Figure 5, the risks of failure from the trapezoidal and wide, triangular fuzzy shapes were greater than those from the narrow triangles and the hat shapes at removal efficiencies greater than the removal goal (> 95%). The extreme values used to define the base of the trapezoid and wide triangle shapes caused the failure area to be large, leading to a larger risk of failure when compared to the narrow triangle and the hat shape. For the same reason, the wide



Figure 5. Comparison of fuzzy shapes for TCE removal. (Removal goal = 95%, S = 4.0)

peak of the trapezoid led to larger failure rates than the wide triangle. Both the hat shape and the narrow triangle resulted in similar possibilities of failure, but the narrow triangle's thinner base led to smaller failure areas then any of the other fuzzy shapes at removals greater than 95 percent. In general, at higher removal efficiencies (i.e., overdesigned systems), the wider the base of the fuzzy number, the greater the risk of failure. Thus, the extreme minimum values and extreme maximum values of a parameter must be chosen carefully. Also, note that differences in the possibilities of failure in Figure 5 are due to the assumptions about the shape, and not the original data or design parameters.

At lower removal efficiencies (i.e., underdesigned systems), as shown in Figure 5, the fuzzy shapes with a wider base (trapezoid and wide triangle) offer lower possibilities of failure, while the narrow fuzzy shapes (hat and narrow triangle) result in larger possibilities of failure. In this case, the narrow triangle gives the largest possibilities of failure, and the trapezoid results in the smallest risk of failure. These results make sense because at design removal efficiencies below the removal goal, the only situations where a system will achieve the removal goal are those that fall on the extreme right (tail) of each fuzzy shape. Since the trapezoid contains more area under its tail as compared to the hat shape, the area of success is larger and the risk of failure is smaller. Note that the fuzzy number shapes shown on Figure 5 do not have a possibility of failure equal to 50 percent at a design removal of 95 percent (the same removal as the removal goal), because the fuzzy numbers created for the uncertain parameters are not symmetric about the median points.

Another comparison was made between the different fuzzy shapes with bromoform as the model contaminant. Figure 6 shows a range of designs for bromoform removal with the stripping factor (2.0), diameter (3.5 m), and flowrate (5.68 m³/min.) held constant. As with Figure 5, only the tower height was varied (8.92 to 19.4 m); therefore, the K_La wa the same for all designs. The removal goal for all of the designs was 95 percent; however, the actual design removal efficiencies varied from 91 percent to 99 percent. Because of bromoform's lower volatility, the tower designs in Figure 6 have larger tower heights, diameters, and A/W ratios than the designs in Figure 5.

The comparison between fuzzy shapes for bromoform removal in Figure 6 is very similar to the comparison between fuzzy shapes for TCE removal (Figure 5), even though the fuzzy Henry's constants of the two compounds are very different. At removal efficiencies greater than 95 percent, the narrow triangle and hat shape have the smallest possibilities of failure, while the wide triangle and trapezoid have the largest possibilities of failure. However, with designs below the removal goal (< 95%), the opposite trend occurs. The thin base of the narrow triangle makes the failure area large in comparison to the total area under the triangle shape, and the large area in the tail of the trapezoid and wide triangle gives those shapes some area of success to keep the risk of failure lower.

An examination of Figures 5 and 6 shows that, at the same design removal efficiency, TCE designs have higher possibilities (risk) of failure than bromoform

FUZZY SET THEORY / 141



Figure 6. Comparison of fuzzy shapes for TCE removal. (Removal goal = 95%, S = 2.0)

designs. There are two reasons for this result, and both are related to the fuzzy Henry's constant of the contaminants. First, the fuzzy Henry's constant for TCE has a larger percentage difference between the extreme minimum and maximum values than bromoform's fuzzy Henry's constant as shown listed in Table 1. This larger uncertainty in the fuzzy Henry's constant of TCE leads to a larger risk of failure. Second, the shape of bromoform's fuzzy Henry's constant is slightly different than that of TCE. There is not a long tail at the lower values for the Henry's constant of bromoform. The difference between the extreme minimum and likely minimum values for bromoform and TCE are 3.1 percent and 29 percent, respectively. This lack of lower values for Henry's constant means that the tower designs for bromoform removal are less likely to fail than those for TCE.

As demonstrated in Figure 5 and 6, the results obtained from an air stripping tower design depend upon the shape of the fuzzy solution. The fuzzy solution to any function of fuzzy numbers takes on the shape of the fuzzy numbers used as inputs, as illustrated in Figure 7. For this reason, the small tails on the base of the hat-shape fuzzy number make it more sensitive at large removal efficiencies than the other shapes. At large removal efficiencies, the removal goal intersects the fuzzy solution at the tail of the hat shape and a small failure area is obtained, leading to a small risk of failure. However, with the same amount of over-design



Figure 7. Fuzzy hat-shaped removal efficiency (A) and trapezoidal removal efficiency (B) from FASAP for TCE. (Removal goal = 98%, S = 4.0, tower height = 9.2 m)

using narrow, triangular fuzzy numbers, the removal goal will not intersect the fuzzy solution and the failure area is zero. While it is true that the trapezoid and the wide triangle have the same base width as the hat shape, the larger size of the peaks in the trapezoid and the wide triangle give larger failure areas (and greater possibility of failure) in comparison to the hat shape.

After comparing the different possible fuzzy shapes, the hat shape was selected for use in the rest of this study. The hat shape was selected, because it contains more information than the other shapes; and, it better represented the raw data [22].

Comparison of Fuzzy and Probabilistic Methods

After a wide range of air stripping tower designs were analyzed, the results were compared with the results obtained by a Monte Carlo approach [1]. The same tower designs (height, diameter, A/W) and assumptions were used to calculate the risk of failure for this study and that of Freiburger et al. [1]. TCE was chosen as the model contaminant and the same parameters were held constant as those noted in the Methodology. The same data used to create the cumulative density functions for the Monte Carlo simulation were used to create hat-shaped fuzzy numbers for the fuzzy simulations.

The results from the comparison between the Monte Carlo and fuzzy set theory (FASAP) approach to simulating system uncertainty are illustrated in Figure



Figure 8. Comparison of fuzzy set theory and Monte Carlo simulations. (TCE removal goal = 95%)

8. The removal goal for all the designs was 95 percent; and, as expected, greater over-designs (removal efficiencies > 95%) offered a lower risk of failure. The results from the different methods of analyzing air stripping tower risk of failure tended to be similar, especially in the failure range of greatest interest (failure rate < 10%). Figure 8 that FASAP tended to estimate a greater risk of failure than the Monte Carlo at lower removals (< 96%), and FASAP tended to estimate a lower risk of failure than the Monte Carlo at higher removals (> 96%). The reason for this result stems from the difference in shape between the normal distribution and the hat-shaped fuzzy number. The normal distribution, which was assumed for the errors on the mass transfer correlations in the Monte Carlo, has infinitely long elongated tails on either side of the large area about the mean. The fuzzy hat-shape used in FASAP also has tails, but they are finite in length.

At a low risk of failure, the lower tail (minimum possible Henry's constant and $K_{L}a$) of the distributions becomes most important and the smaller tail of the hat shape leads to smaller failures areas and fewer possibilities of failure when using FASAP. However, at a high risk of failure, the upper tail (maximum possible Henry's constant and K_1a) becomes most important, and the smaller tail of the hat shape makes the failure area large in comparison to the total area under the curve, which leads to a greater failure risk.

In general, when designing a tower with a design removal efficiency less than the removal goal (underdesign), variability in the parameters is desirable to ensure some possibility of design success. However, when designing at a design removal efficiency greater than the removal goal (overdesign), variability in the parameters is undesirable because it leads to a risk of design failure. Thus, the normal distribution's greater range of possible parameter values compared to fuzzy distributions, leads to less risk of failure for underdesigns and greater risk of failure for overdesigns.

Analysis of Air Stripping Designs Using FASAP

The computational ease of using fuzzy set theory to model the risk of system failure and cost enabled a wide range of system designs to be simulated and compared. A wide range of possible tower heights, tower diameters, and air-to-water ratios that resulted in the same design removal efficiency were used to compare the importance of the actual tower configuration (design). The flowrate (5.68 m³/minute), temperature (10°C), and packing (7.6-cm plastic saddles) were held constant for all simulations. The risk of failure and total cost were determined for each design, and the results from each chemical were compared in order to determine rational air stripping design strategies.

A wide range of tower designs (208) were examined with TCE as the model contaminant in this study. The risk of failure and the system cost were determined for each design; even though multiple designs have the same design removal efficiency, they have different costs and risk of failure (since they have

different mass transfer rates). The relationship between risk of failure and cost for a design life of five years is depicted in Figure 9. The abcissa in Figure 9 represents the possibility (or risk) that the tower does not remove 95 percent of the contaminant, given the uncertainties in Henry's constant and the mass transfer coefficients. The ordinate in Figure 9 represents the total treatment cost including both capital and operating costs. Each symbol represents a tower design with a unique combination of tower height, tower diameter, and A/W ratio. Each group of symbols represents designs with the same design removal efficiency. The design removal efficiency is the removal achieved by the tower when the mean values of Henry's constant and the mass transfer coefficients are assumed.

For each group of designs with the same removal, costs varied widely, but the risk of failure only varied within a small range, as illustrated in Figure 9. As expected, the greater the degree of overdesign (larger design removal efficiency), the lower the risk of failure and the higher the cost. Figure 9 shows that for a small increase in cost, a significant decrease in risk can be achieved. For example, the least expensive design for 95.7 percent removal is slightly cheaper than the least expensive design for 98 percent removal, but the 98 percent removal design is about 35 percent more reliable.

Figure 9 also illustrates that designs with different design removal efficiencies can have a similar risk of failure and cost. The best designs were assumed to be



Figure 9. Range of designs for removal of TCE. (5-Year design life and removal goal = 95%)

those with the least cost and risk of failure (i.e., closest to the origin). Many designs are inefficient, because their total costs are equal to designs with a lower risk of failure. Also, many designs have the same risk of failure as the best designs but are more expensive. Thus, care must be taken when designing the optimal air stripping tower by minimizing the risk of failure and cost, while examining a variety of removal efficiencies.

There is disagreement among researchers [1, 16] about which A/W ratios (higher or lower) yield more reliable designs. The methodology used in this research allowed this issue to be examined. Figure 10 illustrates the relationship between the risk of failure and the A/W ratio for a subset of the designs shown in Figure 9. The risk of failure is plotted versus the total cost of each design and the A/W ratio of each design is shown rounded to the nearest integer. Some designs share the same A/W ratio, but each design is unique and has a different tower height and/or diameter. The designs in Figure 10 are those with a design removal efficiency of 96 percent. Figure 10 shows that the design with the lowest risk of failure has the lowest feasible A/W ratio, in this case 11. This result agrees with the conclusion of Freiburger et al. [1] that the smallest stripping factors yield designs with lower failure rates.

A similar analysis of designs for 98 percent removal efficiency also was performed. Figure 11 depicts designs from Figure 9 with 98 percent design removal efficiencies. Again, the risk of failure is plotted versus the total cost of each



Figure 10. Air-to-water ratios of designs with 96% TCE removal efficiency. (5-Year design life and removal goal = 95%)



Figure 11. Air-to-water ratios of designs with 98% TCE removal efficiency. (5-Year design life and removal goal = 95%)

design and the A/W ratio of each design is shown rounded to the nearest integer. Figure 11 shows that designs with higher A/W ratios (44, 89) have a lower risk of failure. Also, the best design under these conditions utilize an A/W ratio of 44. This A/W ratio is not the largest used in this study, but is four times as large as the A/W of 11, which was considered optimal at 96 percent removal efficiency. This result shown in Figure 11 is in agreement with research by Roberts et al. [16] who stated that a large A/W ratio is needed to minimize the effect of errors in estimating mass transfer. Roberts' conclusion has often been assumed to mean that high A/W ratios will lead to a more reliable tower design.

Further research at other removal efficiencies (95.7, 97, 99, and 99.5%) for TCE indicated that at lower removal efficiencies (\leq 96%) a lower A/W ratio (11) was the best; however, at higher removal efficiencies (\geq 97%), higher A/W ratios (44 and 89) led to more reliable designs. This switch from a lower A/W to a higher A/W ratio yielding optimal designs is referred to as the "A/W ratio switch." This A/W ratio switch was found to occur for TCE and PCE at different removal goals but not for bromoform.

CONCLUSIONS

1. As an alternative to the probabilistic approach, fuzzy logic can be used to develop a risk-based design methodology for air stripping towers.

Furthermore, fuzzy logic can be successfully applied to other technologies where uncertainties exist in key parameters.

- 2. Four shapes were examined for use as fuzzy numbers in this research: wide triangle, narrow triangle, trapezoid, and hat shape. The hat shape allowed the input of more information than the other shapes and was more sensitive at regions where few possibilities of failure are expected.
- 3. Fuzzy logic and probabilistic methods yielded similar results for the same design conditions in this study.
- 4. Probabilistic methods require massive amounts of computer time, assumptions about the distribution of the parameters, and assumptions concerning the independence of uncertainties in each parameter. In contrast, fuzzy logic requires fewer assumptions and requires less computing power than probabilistic methods. Also, fuzzy mathematics account for any possible dependencies that may exist between parameters.
- 5. In addition to cost, the risk of failure of air stripping towers is an important design consideration. When a wide range of designs are examined, it becomes clear that a significant overdesign is both cost-effective and more reliable.

REFERENCES

- E. J. Freiburger, T. L. Jacobs, and W. P. Ball, Probabilistic Evaluation of Packed-Tower Aeration Designs for VOC Removal, *Journal American Water Works Association*, 85:10, pp. 73-86, 1993.
- X. Q. Gui and C. E. Goering, Introduction to Fuzzy Set Theory and Applications, *Transactions in Agriculture*, ASAE, 33:1, pp. 306-313, 1990.
- 3. E. Hisdal, Interpretive Versus Prescriptive Fuzzy Set theory, *IEEE Transactions on Fuzzy Systems*, 2:1, 1994.
- C. C. Tannehill, M. F. Dahab, W. E. Woldt, and I. Bogardi, Evaluation of Nitrate Treatment Methods Under Uncertainty, *Journal of Environmental System*, 25:4, pp. 421-444, 1996-97.
- M. E. Hagemeister, D. D. Jones, and W. E. Woldt, Hazard Ranking of Landfills Using Fuzzy Composite Programming, *Journal of Environmental Engineering*, ASCE, *122*:4, pp. 248-258, 1996.
- M. T. Yin and M. K. Stenstrom, Fuzzy Logic Process Control of HPO-AS Process, Journal of Environmental Engineering, ASCE, 122:6, pp. 484-492, 1996.
- P. Tucker, M. Grant, and J. Lamont, Predicting Recycling Scheme Performance: A Process Simulation Approach, *Journal of Environmental Management*, 53:1, p. 31-49, 1998.
- 8. W. H. Dong and H. C. Shah, Vertex Method for Computing Functions of Fuzzy Variables, *Fuzzy Sets and Systems*, 24, pp. 65-78, 1987.
- 9. A. H. Ang and W. H. Tang, *Probability Concepts in Engineering Planning and Design*, Volume 2, John Wiley & Sons, New York, 1985.
- B. P. Shrestha and L. Duckstein, A Fuzzy Reliability Measure for Engineering Applications, Chapter 7 in *Uncertainty Modeling and Analysis in Civil Engineering*, B. Ayyub (ed.), CRC Press, New York, 1997.

- 11. A. L. Hines and R. N. Maddox, *Mass Transfer Fundamentals and Applications*, Prentice-Hall, Englewood Cliffs, New Jersey, 1985.
- 12. K. Onda, E. Sada, and Y. Murase, Liquid-Side Mass Transfer Coefficients in Packed Towers, *A.I.Ch.E. Journal*, 5:2, pp. 235-239, 1959.
- K. Onda, H. Takeuchi, and Y. Okumuto, Mass Transfer Coefficients Between Gas and Liquid Phases in Packed Columns, *Journal of Chemical Engineering of Japan*, 1:1, pp. 56-62, 1968.
- B. I. Dvorak, D. F. Lawler, J. R. Fair, and N. E. Handler, Evaluation of the Onda Correlations for Mass Transfer with Large Random Packings, *Environmental Science and Technology*, 30:3, pp. 945-953, 1996.
- J. M. Gossett, C. E. Cameron, B. P. Eckstrom, C. Goodman, and A. H. Lincoff, Mass Transfer Coefficients and Henry's Constants for Packed-Tower Air Stripping of Volatile Organics: Measurements and Correlations, *Air Force Eng. & Services Lab. Report No. ESL-TR-85-18*, Tyndall Air Force Base, Florida, 1985.
- R. V. Roberts, G. D. Hopkins, C. Munz, and A. H. Riojas, Evaluating Two-Resistance Models for Air Stripping of Volatile Organic Contaminants in a Countercurrent, Packed Column, *Environmental Science and Technology*, 19:2, pp. 164-173, 1985.
- 17. J. S. Stansbury, *Risk-Cost Analysis Under Uncertainty for Dredged Material Disposal*, doctoral dissertation, University of Nebraska at Lincoln, 1991.
- E. J. Freiburger, Probabilistic Evaluation of Packed-Tower Aeration for the Removal of Volatile Organic Chemicals, masters thesis, Duke University at Durham, North Carolina, 1993.
- T. K. Sherwood and F. A. L. Holloway, Performance of Packed Towers-Liquid Film Data for Several Packings, *Transcripts of the American Institute of Chemical En*gineering, 36:1, pp. 39-70, 1940.
- B. I. Dvorak, C. J. Herbeck, C. P. Meurer, D. F. Lawler, and G. E. Speitel, Jr., Selection Among Aqueous and Off-Gas Treatment Technologies for Synthetic Organic Compounds, *Journal of Environmental Engineering*, ASCE, *122*:7, pp. 571-580, 1996.
- J. Q. Adams and R. M. Clark, Evaluating the Costs of Packed-Tower Aeration and GAC for Controlling Selected Organics, *Journal American Water Works Association*, 83:1, pp. 49-57, 1991.
- 22. J. W. Schauble, *Use of Fuzzy Logic for Quantifying Uncertainty in Air Stripping Design*, masters thesis, University of Nebraska at Lincoln, 1997.

Direct reprint requests to:

Dr. Bruce I. Dvorak W348 Nebraska Hall Dept. of Civil Engineering University of Nebraska Lincoln, NE 68588-0531