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ON HIERARCHICAL STRUCTURES ARISING SPONTANEOUSLY IN MARKETS WITH PERFECT COMPETITION*

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ABSTRACT

It is shown that in the market with the perfect competition there may be certain hierarchical structures arising spontaneously. The elements of these structures are connected with each other by trade interaction between firms which can play the role of buyers and sellers simultaneously and transform material flow to ultimate consumers. All firms minimize efforts to satisfy the consumer demands. The property of ideal supplying of consumers with goods is found in the market with the perfect competition.

In real economic systems there must be a certain mechanism that informs people what they should produce and in what amount, what work should be performed for this production, etc. Broadly speaking, there are two essentially different mechanisms governing economic life. One of them is based on the state hierarchical system, where behavior of each economic agent is directly controlled by one of the higher rank [1]. According to the other mechanism the correlation in people behavior is grounded on the spontaneous order which arises through information obtained by individuals in the interaction with their local economic circumstances. The latter mechanism is actually the core of real markets [1-3].

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In the present article we call attention to the fact that the market can also contain hierarchical structures arising spontaneously where each of their elements responds to the corresponding piece of information, solves its individual problem, e.g., "maximizing" own profit, which, nevertheless, leads to the perfect functioning of the system as a whole. The ideality in the behavior of these systems is caused by self-processing of information at each hierarchy level, i.e., by synergetic mechanism of self-regulation [4].

Let us first discuss the reason and the place where such hierarchical structures come into existence.

The existence of a tremendous amount of goods in market, in contrast to a relatively small number of raw materials as well as producer specialization shows that there must be a highly complex network which links different producers with one another, transforms raw materials and, finally, supplies consumers with the goods required. The structure of such a network is schematically illustrated in Figure 1. A branch of the given network (e.g., branch i) joining the nearest nodes represents a collection of producing firms that can be regarded as identical from the standpoint of their input and output. The firms are linked with one another by sell-buying processes, the output of firms belonging to higher ranks being input for firms of lower ranks. The nodes specify these sell-buying interactions. Firms at the last level sell directly to ultimate consumers, supplying the latter with different types of goods. In other words input and output of different firms form in given network a material flow $\{X_i\}$ going from raw material stems to the consumer medium.

It should be noted that this sell-buying interaction singles out a certain economic system under consideration that on one hand, involves a great number of participants, and exhibits a general property typical for all markets, and, on the other hand, is a small part of the whole market society producing goods of a certain type. For example, steel, food, clothing industries may be regarded as such microeconomic markets. In living organisms, a regional vascular network of different organs play the same role, as such microeconomic markets in the human society. In these terms the material flow corresponds to the blood flow in vessels and the consumer medium is related to the cellular tissue.

Particular interconnections between different firms can occur and disappear during formation and evolution of the market under consideration. These interactions are governed by trade with each other. The latter process stimulated the money flow within the market network in the direction opposite to the material flow, i.e., in the direction from the consumers to the producers of raw materials. The conservation of money at the nodes enables them to play a role of a certain aggregated information of the state of the consumer medium as well as the firm activity. The matter is that for a certain collection of firms, e.g., firms, belonging to branch i, to be able to supply firms of the lower rank linked directly with the given firms of the network with the required input it is necessary and sufficient that these firms possess the information characterizing the consumer state in the

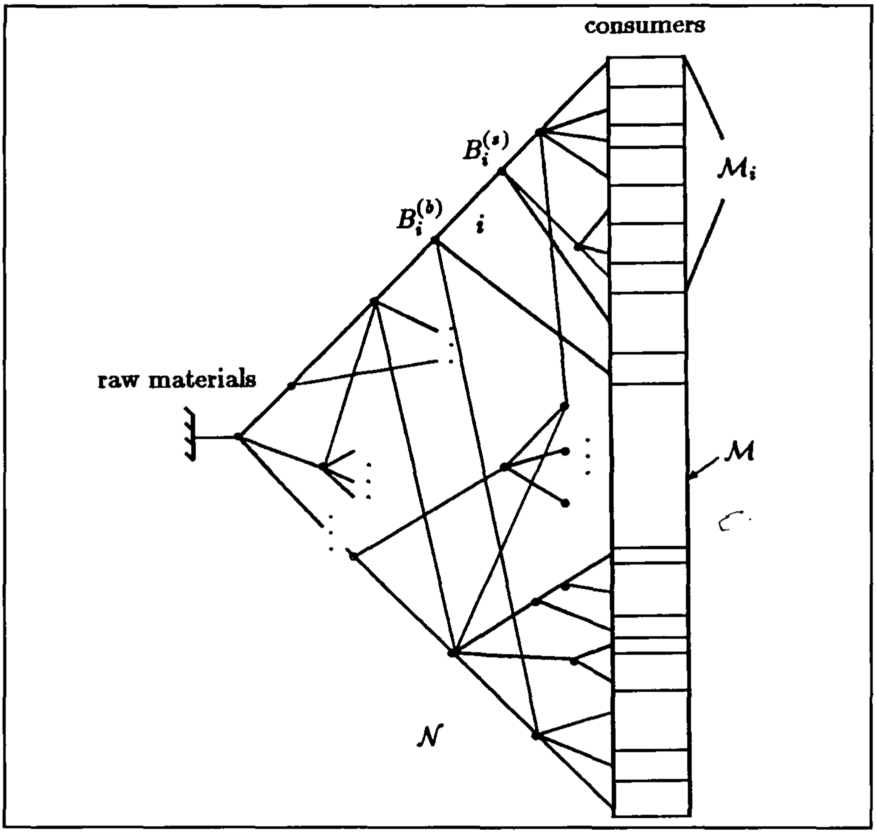

Figure 1. Schematic representation of the market network.

region controlled by the given firms as a whole. Such information is directly reflected in the price of its output.

A change in the consumer demand leads to variation of the material flow within the market network. The latter, in turn, causes the firm's profits to vary and, thus, the firms to increase or decrease their activities. In particular, there are no barriers to the entry of new firms in respect to the short-run profits being made in the given market. When competition is perfect this process will cause the average profit at each branch to be maintained at zero value [2, 3].

In the present article within the framework of the market with perfect competition we show that there may be a certain hierarchical structure arising spontaneously that supplies consumers with goods ideally. This means that at the first approximation change in demand at one point of the consumer medium does not

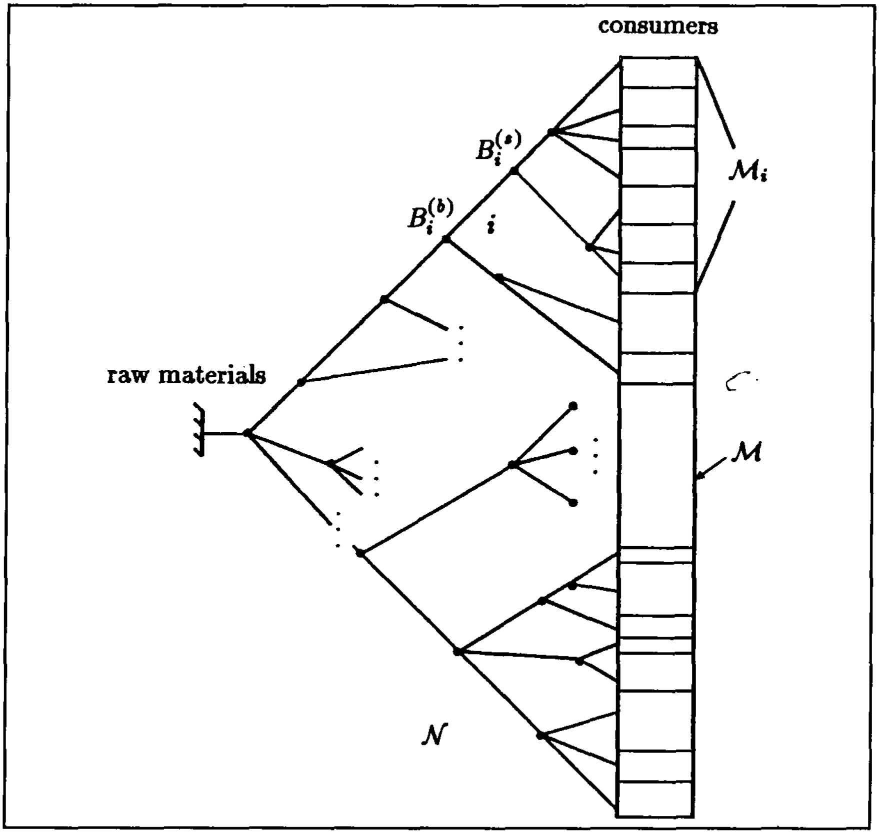

Figure 2. Industry structure of the tree form.

cause variations in the goods flow at its other points although the material flow varies across all branches belonging to all the hierarchy levels.

Let us begin by setting up the model.

MODEL

Let us consider an industry structure in the market involving consumer medium M and hierarchical network N of the tree form supplying it with goods (see Figure 2).

Material flow in the given network is determined by the collection $\{X_i\}$ of the total firm product X_i at branch i measured in mass units. Because of the conservation of materials at each node, e.g., node B, we may write the expression

$$(X_i)_{in} = \sum_{jB} (X_j)_{out} \tag{1}$$

where $(X_i)_{in}$ and $(X_j)_{out}$ are the total product at the branches going in and out of the node B and the sum runs over all the branches leaving this node. In other words, the output $(X_i)_{in}$ of firm i is equal to the sum of the inputs of the firms $(X_j)_{out}$.

The branch i is assumed to contain n_i individual firms treated as identical, at least, on average. The total product X_i at the branch i is equal to sum of the products x_i produced by these n_i firms, viz.:

$$X_i = n_i x_i \tag{2}$$

Variations in the number of firms $\{n_i\}$ is the market response to change in the consumer demand. The output collection $\{X_i\}$ of the firms belonging to the last level of the network determines the corresponding set of goods flows $\{X_i^*\}$ through the consumer medium:

$$X_i = X_i^*$$

For the sake of simplicity we suppose that i-th consumer is supplied by only one of the firms at the last level. The latter can be justified if the consumers are substantially distinguished by their location in physical or goods space.

The trade interaction at a node B gives rise to a price P_B for a mass unit of the material exchanged in this interaction. As a result, the money flow aggregated at branch i due to trade interaction is $X_i(P_i^{(s)} - P_i^{(b)})$, where $P_i^{(s)}$, $P_i^{(b)}$ correspond to the nodes $B_i^{(s)}$, $B_i^{(b)}$ at which firms i play the role of seller and buyer respectively (Figure 2). The individual profit of a firm belonging to branches i is

$$\pi_i = x_i(P_i^{(s)} - P_i^{(b)}) - tc_i(x_i) \tag{3}$$

where the former term is its revenue and the latter one is its total cost given by the expression

$$tc_i(x_i) = k_i + a_i x_i + b_i x_i^2 \tag{4}$$

The set of parameters $\{k_i, a_i, b_i\}$ are considered to be given constants beforehand. The total profit Π_i at branch i is

$$\Pi_i = X_i(P_i^{(s)} - P_i^{(b)}) - [k_i n_i + a_i X_i + \frac{1}{n_i} b_i X_i^2]$$

The individual purpose of each firm is to maximize its profit with respect to the product, which leads to the expression

$$\frac{\partial \Pi_i}{\partial X_i} = 0 \tag{5}$$

whence it follows that

$$P_i^{(s)} - P_i^{(b)} = a_i + \frac{2}{n_i} b_i X_i \tag{6}$$

At the final stage firms sell their goods to ultimate consumers, accordingly, price $P_i^{(s)}$ is determined by the consumer market's demand. Assuming that at this final stage $P_i^{(s)} = d_i^* - f_i^* X_i^*$, where d_i^*, f_i^* are supposed to be predetermined constants and the firms transform each unit of their input into one unit of the output after a processing cost of c_i^* per unit is incurred [2] we obtain.

$$\Pi_i^*(X_i^*) = X_i^* \, (d_i^* - f_i^* X_i^*) - P_i^{(b)} X_i^* - c_i^* X_i^* \tag{7}$$

The condition of the $\Pi_i^*(X_i^*)$ attaining maximum with respect to X_i^* leads to the expression

$$P_i^{(b)} = d_i^* - c_i^* - 2f_i^* X_i^* \tag{8}$$

Perfect competition in the market maintains the profit $\Pi_i(X_i)$ at the zero value [3], thus, for each branch i of the network it can be written

$$\Pi_i(X_i|P_B)| = 0 \tag{9}$$

for the $\{X_i\}$ and $\{P_B\}$ related to each other by expressions (6), (8).

Expression (6), (8) together with the conservation (1) of products at the nodes establishes such prices at the nodes that meet the maximum profit condition for all the firms and satisfy the market equilibrium.

Now let us analyze the characteristic properties of the stated model.

PERFECT SELF-REGULATION

Substituting (6) into (9) we find that

$$P_i^{(S)} - P_i^{(b)} = a_i + 2\sqrt{k_i b_i} \tag{10}$$

This result shows that the difference $P_i^{(s)} - P_i^{(b)}$ for each branch i is actually determined by the internal parameters of the technology, production efficiency, and the market rate of capital rather than by the consumer demand. The latter follows directly from the perfect competition. This property enables us to find the price at a given node. Setting for the sake of simplicity for firms belonging to the stem the buyer price be equal to zero (or ignoring it) we get for a node B

$$P_B = \sum_{i \in P_B} [a_i + 2\sqrt{k_i b_i}\,] \tag{11}$$

where P_B is a path on the network connecting the given node B with the stem.

Returning to the initial network shown in Figure 1, we can replicate the same speculations with respect to material flow distribution $\{X_i\}$. In this way we again will obtain formula (11) where, however, the path P_B is not unique. Therefore, except for the degenerate cases, under perfect competition the firms that belong to the paths (fine lines on Figure 1) with larger values of P_B have to leave the market. The latter will convert the initial production network of complex geometry into a hierarchical network of the tree form (solid line in Figure 1), which minimizes the price.

According to (8) and (11) the goods flow X_i^* demanded by consumer i can be given as

$$X_i^* = \frac{1}{2f_i^*}[d_i^* - c_i^* - \sum_{j\varepsilon P_i}(a_j + 2\sqrt{k_j b_j})] \tag{12}$$

where P_i is the path leading from the stem to the given point i of the consumer medium. The state of the consumer medium is specified entirely by the set of parameters $\{d_i^*, f_i^*\}$. Thus, change in the consumer demand reflects in time variations of the parameters $\{d_i^*, f_i^*\}$. Assuming the technological parameters $\{a_i, k_i, b_i, c_i^*\}$ of the production to be constant it can be seen from (12) that the goods flow X_i^* through any given point i of the medium M is controlled solely by its own parameters $\{d_i^*, f_i^*\}$. Time variations of the parameters $\{d_i^*, f_i^*\}$ at other points do not affect the goods flow at the given point i. This property can be naturally treated as perfect self-regulation.

Conservation (1) of materials at the nodes allow us to find the material flow X_i going through a given branch i as a function of the goods flow through the consumer medium. Specifically,

$$X_i = \sum_{j\varepsilon M_i} X_j^* \tag{13}$$

where M_i is the consumer medium region where the goods flow as a whole is directly controlled by the given branch (Figure 2). In particular, expressions (6), (10), (13) yields the number n_i of independent firms participating in the production of the output X_i:

$$n_i = \sqrt{\frac{bi}{ki}} \sum_{j\varepsilon M_i} \frac{1}{2f_j^*}\left\{d_j^* - c_j^* - \sum_{j'\varepsilon P_j}(a_j' + 2\sqrt{k_j' b_j'})\right\} \tag{14}$$

The last expression reflects the ability of the market as a whole to respond to changes in the consumer demand by appropriate change in the number of firms n_i.

REMARKS

An opinion exists that the market economies are based solely on spontaneous trade interactions between buyers and sellers in contrast to the centrally-planned economies which are organized hierarchically. In the present article we tried to show that "free markets" might also contain complex hierarchical structures, which, at first, arise spontaneously, and then minimize efforts to satisfy the consumer demands. Such hierarchical systems are likely to provide the unique feasibility of self-processing of information on which products to supply, how much of them to produce, and in what ways to distribute them.

Naturally, real markets are not perfect, firms are not identical, and so on. This leads to violations of the market ideal self-regulation. Nevertheless, we think the presented model may be useful in analyzing real processes in markets if used as a first approximation, since it enables one to directly take into account possible complex interactions between producers.

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