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SELECTION OF HAZARDOUS WASTE DISPOSAL ALTERNATIVE USING MULTI-ATTRIBUTE UTILITY THEORY AND FUZZY SET ANALYSIS

G. ANANDALINGAM

MARK WESTFALL

Department of Systems
University of Pennsylvania, Philadelphia

ABSTRACT

We examine the problem of choosing a hazardous waste disposal alternative, where there are multiple attributes associated with each site, and the decision maker has imprecise information regarding the impact of the waste on each site, and his preference ordering for each attribute. We show how developments in multi-attribute utility theory (MAUT) and fuzzy set analysis can be used to address the problem. We apply the methodologies to a case study concerning the disposal of PCB contaminated transformer fluids at one of three sites, and conclude that MAUT is superior to fuzzy set analysis in cases where there is imprecision in either value scores or preference weights, but not both.

INTRODUCTION

Industrial chemicals are often quite toxic and could cause allergies, damage vital organs of the human body, and cause and promote cancer. According to the Office of Technology Assessment [1], American industry generates approximately 255 to 275 million metric tons of industrial chemical wastes annually. Much of the industrial waste is dumped in landfills and could be quite hazardous. Greenberg and Anderson have concluded after an extensive study that many of the landfill sites are unmonitored and the wastes leach into the groundwater, causing significant, and potentially catastrophic, contamination [2].

Landfills are not the only technique for disposing of the hazardous waste. There are basically three strategies for solving the problem: the wastes can be

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reduced at the source, converted to less hazardous or non-hazardous substances, or placed as residuals in the environment. Waste reduction consists of four components: abatement, minimization, reuse, and recycling. (See [3] for detailed description of these options.) Incineration and biotechnological treatment detoxify wastes and are examples of conversion to less hazardous substances. Landfills, surface impoundments, and underground injection wells are examples of placing the residuals in the environment. Each of these technologies has its merits and drawbacks.

The choice of waste disposal alternative has been a challenge to the governmental decision makers. The main problem of locating landfills, for instance, have been characterized by the phrases LULU (Locally Unwanted Land Use) and NIMBY (Not In My Back-Yard), reflecting community responses (or opposition) to the selection of sites. The community opposition has to be incorporated into any decision concerning waste disposal technology. In addition, the alternatives have to be evaluated on other criteria including air and water quality, land use, demography, emergency response, technical and economic parameters, etc. Issues involve what relative value should be placed on each criterion, how to incorporate the value judgements of the affected community into the decision, and how qualitative and quantitative information should be combined.

In this article we examine the problem of choosing a hazardous waste disposal alternative using developments in multi-attribute utility theory (MAUT) and fuzzy set analysis. The basic MAUT model involves obtaining values scores for each alternative for each of multiple attributes (criteria) and then combining the scores by weighting them by scaling constants that specify the importance of each attribute. In most cases, decision makers have difficulty responding to the lottery based questions that result in value scores and preference weights [4]. Thus, the methodological focus of this article is on making decisions when there is only imprecise information.

We present MAUT-based methodologies for making decisions when some information is given in ordinal terms (for example, preference order among attributes) and some in cardinal terms. When the decision maker can present semantic, degree of preference, or rating information, we use developments in fuzzy set analysis to solve the problem. These methodologies are applied to a case study of a utility company considering the disposal of PCB-contaminated transformer fluids using one of three incineration alternatives. Besides making the decision on the optimal site, we also examine the relative merits of MAUT-based and fuzzy set methodologies in addressing the problem of imprecision. We find MAUT based methodology to be superior in situations where there is imprecision in either the value scores or the preference weights but not both. Fuzzy set analysis seems to perform better when there is complete imprecision.

The article is organized as follows: The next section develops the MAUT and fuzzy set methodology to be used in this study. Section 3 describes the hazardous waste disposal problem and presents the salient information. Site selection computations are presented in Section 4 with discussions on the results. The final section assesses these results.

2. MULTI-ATTRIBUTE DECISION MAKING UNDER IMPRECISE INFORMATION

Overview

We are considering a problem where the ultimate decision is based on the assessment of impacts on multiple criteria or attributes, $X_1, X_2, \ldots, X_n$. In order to facilitate the decision, we invoke an evaluation (or value) function $v(x_1, x_2, \ldots, x_n)$ where x_i is the specific level of X_i, that has the property that $v(x_1^A, x_2^A, \ldots, x_n^A) > v(x_1^B, \ldots, x_n^B)$ when alternative A is preferred to B. (Here x_i^j is the level of X_i for alternative j.) The overall preferability function $v(\cdot)$ is often decomposed into the individual components by using the weighted-additive form:

$$v(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} k_i \; v_i(x_i), \tag{1}$$

where the v_i are single-dimensional value functions, and k_i are positive weighting (scaling) constants (which can be normalized by imposing $\Sigma k_i = 1$). The additive value structure satisfies axioms of rational decision making [5], and is the most common form used in applications (see [6-8], for example).

In order to use equation (1) for real world decision making problems, it is important to obtain specifications of k_i, the preference weights, and $v_i(\cdot)$, the individual value functions. Most applications use *cardinal* specifications of these parameters. (Keeney and Raiffa show how to obtain this information [4].) However, in most applications the decision makers are only able to specify this information incompletely. The imprecision in value scores and trade-off weights can be described by *set inclusion* [9], a special case being purely ordinal information. Conversely, the entire problem can be cast in the framework of fuzzy set analysis where we use *membership functions* to specify the degree to which the weights and value scores belong to the set of possible values for these parameters.

We examine three possible degrees of completeness of information:

a. known value scores for all attributes, and imprecise preference structure;
b. known preference weights and imprecise ranking of alternatives by attributes; and
c. imprecise ranking for both values and preference weights.

In the case of (a.), we use concepts initially attributed to Fishburn [10], and later developed by Hannan [11] and Kirkwood and Sarin [12] to solve the problem. In the case of (b.), we use an extension of a ranking method proposed by Cook and Seiford [13]. Finally, when there is only ordinal information, we use fuzzy set analysis developed by Yager [14] for the solution. We now briefly describe these methods.

Cardinal Values and Imprecise Preference Weights

Suppose we know the value scores of each attribute for each alternative: i.e., all $v_i(x_i^j)$ for all i, i = 1, . . . , n, attributes and j, j = 1, . . . , m alternatives are known. In addition, suppose we know the exact ordering of the decision maker preference for the attributes. Without loss of generality, we can assume that this order is given by $k_1 > k_2, \ldots, > k_n > 0$.

Then, it has been proven in different, but related form, by Fishburn [10], Hannan [11], and Kirkwood and Sarin [12], that alternative a is a preferred to alternative a′ if and only if:

$$\sum_{i=1}^{\ell} v_i(x_i^a) \geqslant \sum_{i=1}^{\ell} v_i(x_i^{a'}), \;\; \ell = 1, \ldots, n, \tag{2a}$$

with at least one of the inequalities being strict, i.e.,

$$\sum_{i=1}^{\ell} v_i(x_i^a) > \sum_{i=1}^{\ell} v_1(x_i^{a'}), \text{ for at least one } \ell = 1, \ldots, n. \tag{2b}$$

By invoking equation (2) we can pair-wise rank all alternatives. Also make note of the fact that equation (2) is general and allows pair-wise ranking even when

a. there is indifference in the preference order among some attributes;
b. there is only partial ranking; and
c. there are parallel rankings, etc.

(See [12] for corollaries to the result established by equation (2).)

Suppose the preference weights can be specified more precisely using natural language measures that give degrees of importance. One such set of measures that compare pairs of attributes is given in Table 1. Using these "fuzzy" measures, Saaty has developed a procedure for obtaining a cardinal ratio scale for the attributes compared [15]. The procedure, called the Analytic Hierarchy Process, is described below.

Suppose we are comparing attribute i with attribute j, we assign values a_{ij} from Table 1. For instance, if the attribute 1 is strongly preferred to attribute 3, we assign $a_{13} = 5$. Then we proceed as follows:

1. let $a_{ji} = 1/a_{ij}$ (3.1)
2. let $a_{ii} = 1$ (3.2)
3. construct matrix $a = \{a_{ij} \mid i = 1, \ldots, n;\; j = 1, \ldots, n\}$ (3.3)

Table 1. Linguistic Measures of Preference

Intensity of Importance	*Definition*
1	Equal Preference or Indifference
3	Weak Preference
5	Strong Preference
7	Demonstrated Preference
9	Absolute Preference
2,4,6,8	Intermediate Values

Source: Saaty [15].

Saaty has shown that the eigenvector corresponding to the maximum eigenvalue of A is a cardinal ratio scale for the attributes compared [15]. The eigenvalue problem is solved by

$$Ak = \lambda_{max}.k\,, \tag{4}$$

where λ_{max} is a scalar corresponding to the maximum eigenvalue, and the eigenvector k corresponding to λ_{max} gives the preference weights.

This procedure allows imprecise fuzzy specifications of preferences to be translated into a consistent vector of preference weights which can be used together with equation (1) to obtain the preferred alternative.

Imprecise Value Scores and Cardinal Preference Weights

Now, let us suppose that estimates of the preference weights can be obtained from the decision maker using procedures outlined in the literature on decision theory [10]. However, the attribute value scores are not known exactly for the alternatives. What is known about the alternatives is how they measure up in satisfying the attributes (objectives). So, for example, in the case of the water quality attribute, we may have information on which of the waste disposal sites affects it most, next, least, and so on. For each attribute, the decision maker would provide a rank order as to how the alternatives would satisfy them. Thus, if a_i^j stands for the rank order of the i-th alternative in satisfying the j-th attribute, for the k-th alternative we have

$$A^k = a_1^k, a_2^k, \ldots, a_m^k \;, \; k = 1, \ldots, n\,. \tag{5}$$

Cook and Seiford examine a problem where consensus has to be reached among m decision makers who rank n alternatives separately [13]. They propose a distance measure to obtain a consensus ranking and show that this distance measure satisfies a number of axioms of social justice. This distance measure has been tried in some applications [16, 17], and can be adopted for this problem.

Each attribute in our problem can be considered synonymous with each decision maker in the Cook and Seiford (hereafter C-S) study [13]. In this case, the consensus ranking of alternatives is given by the rank c_i which minimizes the distance measure in the C-S study:

$$D = \sum_{j=1}^{n} \sum_{i=1}^{m} | a_i^j - c_i | . \tag{6}$$

In our study, there is a further complication: each attribute has a different degree of importance given by the cardinal preference weights k_i. Thus, we need to modify the above and obtain a ranking which minimizes

$$D = \sum_{j=1}^{n} \sum_{i=1}^{m} k_i \, | a_i^j - c_i | . \tag{7}$$

As noted by Cook and Seiford, when there are no ties in the rankings, the rank order for the alternatives can be obtained by solving the following assignment problem [13]:

$$\text{Min} \sum_{i=1}^{m} \sum_{i=1}^{m} d_{ij} \; x_{ij} \tag{8}$$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \forall j \tag{9}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \forall i \tag{10}$$

$$x_{ij} \geqslant 0 \quad \forall i,j \tag{11}$$

and

$$d_{ij} = \sum_{\ell=1}^{n} k_\ell \; | \; a_i^\ell - j \, | . \quad \forall j \tag{12}$$

Thus, once the impact of the different alternatives for each attribute is given in ordinal terms, the problem given by equations (8) through (12) is solved for the given cardinal preference weights to obtain an assignment of a different overall rank for each alternative.

The impact of the different alternatives for each attribute can also be given by *membership functions* rather than ordinal measures. The membership function is derived from Zadeh's set theory [18], and represent numerically the degree to which an element belongs to a set. This function takes on values between 0 and 1, and it is an extension of the idea of a characteristic function of a set. Since we have $A = \{a_1, \ldots, a_m\}$ alternatives, a fuzzy subset B of A is characterized by a membership function $U_B(A)$ which associates with every member of A, a number in the interval [0,1] which indicates the grade of the membership of B in A. The fuzzy subset B can be, for instance, the level of water pollution caused by the alternative waste disposal sites.

Now, suppose we have more than one fuzzy set over the alternative A; i.e., more than one attribute over which the alternatives are evaluated. Let us call these attributes $X_1, \ldots, x_n$. In order to evaluate the alternatives, Zadeh suggests using the rule of implied conjunction [18], stated as

$$D = B_1(x_1) \cap B_2(x_2) \cap \ldots \cap B_n(x_n), \qquad (13)$$

where D is the decision, and B_i ( ) is a fuzzy subset of the set of potential alternatives whose membership function indicates how well each of the alternatives satisfies that attribute (or objective). In addition, D is also a fuzzy subset of the set of potential alternatives whose membership function $u_D(A)$ indicates how well each of the candidates satisfies the set of objectives and is given by

$$u_D(A) = \text{Max} \quad \text{Min}[u_{x_1}(a), u_{x_2}(a)] \qquad (14)$$

for all $a \in A$ [18].

Thus, in the case where the value scores are known imprecisely, the procedure is to establish membership functions for each alternative vis-a-vis each attribute, then to derive the membership function for the fuzzy decision using equation (14). Up to this point we have not considered the relative importance of each attribute, but this can be incorporated into the fuzzy set analysis. Zadeh, for instance, associates an operation of raising the set A to its second power (i.e., ^{2}) with the linguistic modifier "very" [18]. For example, if A stands for the attribute "clean," A^2 would stand for "very clean." Yager uses this concept to incorporate scaling (preference) weights as follows [14]:

Definition – Let B be a fuzzy subset over A and let $\alpha \geqslant 0$ be a scalar. The operation of raising B to the α power, denoted by B^α is defined as a fuzzy set over A with membership function:

$$u_{B^\alpha}(A) = [u_B(A)]^\alpha \qquad (15)$$

for all $a \in A$.

Thus, the decision with imprecise value scores and cardinal preference weights are obtained by deriving the membership function for the decision as

$$U_D(A) = \text{Max} \left\{ \text{Min} \left(\; u_{x_1}(a)\right]^{k_1}, \ldots, [u_{x_n}(a)]^{k_n} \right\}. \qquad (16)$$

Thus, equation (16) provides a complete derivation of the multi-attribute decision problem with imprecise value scores.

Imprecise Value Scores and Attribute Preference Weights

In the case where both the value scores and preferences for the alternatives with respect to each attribute is only known imprecisely, we would use a combination of the developments in the previous sections.

Let us consider the case when the values and preferences are given by ordinal measures. Hannan [11] proposes a modification of the Cook and Seiford [13] method for solving this problem.

First, consider the matrix $D = \{d_{ij} \mid j = 1, \ldots, m\}$ using equation (12). Note that this matrix will be given in terms of the preference weights k_ℓ ($\ell = 1, \ldots, n$), and would represent the value scores for each alternative for each priority rank. Next, we need to decide how to rank the alternatives given this information. Here we form value scores in terms of the k's for different combination of priority ranks $R_i(k)$. Since there are m alternatives, there would be m! possibilities (i.e., $i = 1, \ldots, m!$). Finally, we use the knowledge on the ordinal ranking of the k's to obtain the non-dominated ranking of the alternatives.

While this method is likely to work for some number of alternatives, when m is large, m! possibilities have to be assessed, and this could become quite tedious. Conversely, we could follow White, et al. and solve linear programs of the form [9]:

$$\Delta R_{ij}^* = \text{Min } R_i(k) - R_j(k) \quad i \neq j \tag{17}$$

$$\text{s.t.} \quad k_1 > k_2 > \ldots > k_n, \tag{18}$$

where non-dominated ranking of alternatives i would have $\Delta R_{ij} > 0$ for $i \neq j$.

Suppose there is fuzzy specification of the value scores and preference weights which is more informative than purely ordinal ones. Then the procedure follows developments in fuzzy set analysis given in the previous sections. Attribute weights are obtained by getting the decision maker to make responses as in Table 1, deriving the A matrix through equation (3) and then estimating the eigenvector using equation (4). The alternatives' contributions towards satisfying the attributes are characterized by membership functions $u_{xi}(a)$, and the alternatives are ranked (fuzzily) using the k-values derived before and invoking equation (14). Once the imprecise information is provided in the form amenable to fuzzy set analysis, the method is fairly straightforward.

3. THE HAZARDOUS WASTE DISPOSAL PROBLEM

We now examine a case where a utility company is considering the disposal of PCB-contaminated transformer fluids at one of three fossil fuel fired generating plants (for details see [19]). All the plants need modifications to facilitate the disposal of the hazardous waste, but only one can be chosen due to cost constraints.

Table 2. Site Selection/Evaluation Criteria

Criterion	*Characteristic*
Air Quality	Dispersive capabilities of site/plant combination and degree to which waste emissions could concentrate onsite or offsite
Surface Water Quality	Potential for surface water degradation due to spills associated with handling and storage of waste
Groundwater Quality	Potential for groundwater degradation due to spills associated with handling and storage of waste (includes leaching into aquifer)
Ecology	Potential impact on ecological resources of area due to routine operations or emergency conditions
Aesthetics	Visual impacts of hazardous waste operations, including handling, storage, and disposal
Demography	Potential long term exposure to emissions due to routine operations or emergencies
Land Use	Compatibility of the surrounding land use with the hazardous waste operation
Emergency Response	Ability of a response team to combat an emergency associated with a spill or other exposure
Transportation	Distance through which the waste should travel to get to site
Opposition	Political or other organized intervention or opposition to the hazardous waste operation

Source: Horsak and Damico [19].

The plants are to be evaluated on the basis of ten attributes. Table 2 provides a list of these criteria and a description of why they are important for the hazardous waste disposal problem. The criteria themselves were rated by twenty-seven individuals who were chosen as a representative cross section of domain specific experts. These preference weights (scaling constants) are given in Table 3. Numerical ratings for the degree by which each alternative satisfies each criterion is given in Table 4.

The information given in Tables 3 and 4 is subject to the following considerations:

1. the data on preference weights were obtained by getting the experts to respond in a "fuzzy" semantic way using definitions given in Table 1, then estimating the eigenvector corresponding to the maximum eigenvalue as

Table 3. Rating of Criteria

Criterion	*Rating*	*Order of Importance*
1. Air Quality	0.161	1
2. Surface Water Quality	0.156	2
3. Groundwater Quality	0.148	3
4. Ecology	0.115	4
5. Aesthetics	0.111	5
6. Demography	0.106	6
7. Land Use	0.074	7
8. Transportation	0.052	8
9. Emergency Response	0.046	9
10. Opposition	0.031	10

Source: Horsak and Damico [19].

given by equation (4), and finally normalizing the eigenvalue to obtain the weights; thus, we can consider the information in Table 3 at best as "fuzzy," but as reflecting the true ordinal preference structure; and

2. a similar argument is made for the rating of how the alternative sites satisfy the criteria (attributes); thus, the data given in Table 4 are not value scores in the Raiffa-Keeney sense [9], and are fuzzy ratings which, again, would reflect true rankings of the alternatives.

4. SELECTION OF WASTE DISPOSAL SITES

Siting Decision With Cardinal Values and Imprecise Preferences

First, make note of the fact that besides making the decision on the optimal site for disposing the hazardous waste, we are also examining the relative merits of the decision theory and fuzzy set methodologies for this case.

From Table 3 we see that the Air Quality attribute has the largest weight and Opposition has the lowest weight. We select the "best" (i.e., non-dominated) hazardous waste disposal site, by using the methodology developed on pp. 74-75. First we need to order the attributes according to decreasing preference, and next obtain the sum of value scores as in equation (2). Table 5 gives details of the calculation. Clearly we see that the value sum for plant A dominates everywhere the value sum for the other alternatives. Thus, the optimal choice is plant A.

Table 4. Rating of Alternatives for Each Criterion (Ordinal Ranking in Parenthesis)

	Alternative Plants		
Criterion	*A*	*B*	*C*
1. Air Quality	0.9	0.7	0.3
	(1	2	3)
2. Surface Water Quality	0.8	0.9	0.2
	(2	1	3)
3. Groundwater Quality	1.0	1.0	1.0
	(1	1	1)
4. Ecology	0.9	0.9	0.2
	(1	1	2)
5. Aesthetics	0.8	0.9	1.0
	(3	2	1)
6. Demography	1.0	0.5	1.0
	(1	2	1)
7. Land Use	0.8	0.6	0.2
	(1	2	3)
8. Transportation	0.8	0.5	0.2
	(1	2	3)
9. Emergency Response	1.0	0.6	0.3
	(1	2	3)
10. Opposition	0.5	1.0	0.3
	(2	1	3)

Source: Horsak and Damico [19].

Now we will use the ratings given in Table 3, which were obtained by normalizing the eigenvector corresponding to the largest eigenvalue (which is the fuzzy set methodology described on pp. 74-75 and equation (4)). The *aggregated value* scores for the three alternatives then are:

Plant A = 0.837, Plant B = 0.787, Plant C = 0.516.

Here, too, plant A would be the optimal site. Thus, whether purely ordinal weights are considered or whether linguistic responses are transformed using fuzzy set techniques into cardinal weights, the site selected would be plant A. For this case it seems that techniques based on decision theory which require just ordinal information would be superior because of the lower information requirements.

Table 5. Site Selection: Cardinal Values—Imprecise Preferences

	Attributes[a]									
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Plant A										
Value Scored	0.9	1.0	1.0	1.0	0.9	0.8	0.5	0.8	0.8	0.8
Value Sum		1.9	2.9	3.9	4.8	5.6	6.1	6.9	7.7	8.5
Plant B										
Value Score	0.7	1.0	0.6	0.5	0.9	0.9	1.0	0.5	0.6	0.9
Value Sum		1.7	2.3	2.8	3.7	4.6	5.6	6.1	6.7	7.6
Plant C										
Value Score	0.3	1.0	0.3	1.0	0.2	0.2	0.3	0.2	0.2	1.0
Value Sum		1.3	1.6	2.6	2.8	3.0	3.3	3.5	3.7	4.7
Optimal Choice: Plant A										

[a] Note: See Table 3 for description of attribute.

Siting Decision With Imprecise Values and Cardinal Weights

Although the preference weights (scaling constants) given in Table 3 were obtained thorugh the maximum eigenvector method proposed by Yager [14], we will assume that they reflect true decision maker preferences precisely (i.e., not fuzzily). The value scores are not given, instead the order in which each alternative satisfies each attribute is specified (see Table 4). In order to select the optimal site, we use the Cook-Seiford [13] method described on pp. 75-77.

The "decision matrix" D is obtained by estimating its elements d_{ij} from equation (12). Table 6(a) presents this matrix along with a sample calculation for d_{11}. The assignment linear program (equations (7) through (11)) yields the priority assignment as in Table 6(b). The optimal priority vector is [A B C] in that order; i.e., plant A is the optimal site.

In order to use the fuzzy set methodology, we derive weighted values using equation (14) and the fuzzy ratings given in Table 4. (Table 7 gives the weighted value estimations.) The optimal fuzzy decision is (using equation (15)), plant A with membership function 0.966 which shows that the optimal decision is almost precisely that. Given cardinal weights for the preferences among the attributes, the decision would be the same, in this case, whether ordinal value

Table 6. Ordinal Values

(a) Decision Matrix

		Priority		
		1	*2*	*3*
Alternatives	1	0.409	0.924	1.591
	2	0.550	0.450	1.450
	3	1.115	0.885	0.845

$$d_{11} = |1-1| \times 0.161 + |2-1| \times 0.156 + |1-1| \times 0.148 + |1-1| \times 0.115 + |3-1| \times 0.111 + |1-1| \times 0.106 + |1-1| \times 0.074 + |1-1| \times 0.052 + |1-1| \times 0.046 + |2-1| \times 0.031 = 0.409$$

(b) Optimal Assignment

		Priority		
		1	*2*	*3*
Alternative	1	1	0	0
	2	0	1	0
	3	0	0	1

scores are provided or whether fuzzy ratings are extracted after further investigation (or interviews) with the decision makers. As in the previous section, the method based on decision theory seems superior due to the reduced information requirement.

Siting Decision with Imprecise Values and Preference Weights

We should note at the outset that the calculations reported in the previous section were based on fuzzy responses for both values scores (which were given as ratings) and preference weights (which were initially given by semantic responses, then translated into the normalized maximum eigenvector). Thus, the fuzzy set result based on imprecise values and preferences is exactly the same as in the previous section: the optimal site is plant A.

Table 7. Weighted Value

Criterion	Alternative A	Alternative B	Alternative C
1. Air Quality	0.983	0.944	0.824
2. Surface Water Quality	0.966	0.984	0.778
3. Groundwater Quality	1.000	1.000	1.000
4. Ecology	0.988	0.988	0.831
5. Aesthetics	0.976	0.988	0.774
6. Demography	1.000	0.929	1.000
7. Land Use	0.983	0.963	0.888
8. Transportation	0.988	0.965	0.920
9. Emergency Response	1.000	0.977	0.946
10. Opposition	0.979	1.000	0.963
Minimum	0.966	0.929	0.774

In the case of the decision theoretic methodology we use value scores and preferences, given by the ordinal information in Tables 3 and 4. The decision matrix is calculated in terms of the k's using equation (12) and is presented in Table 8. The aggregated value scores for the different priorities for the alternatives are given in Table 9. Using this and the fact that the priority ranking of preferences (Table 3) is $k_1 > k_2 > \ldots > k_{10}$, we find that the best (non-dominated) priority vector is IV which gives plant C as the optimal site. Plant A comes second.

This is quite surprising since all analysis until now has resulted in plant A being optimal and indicated that plant C is the worst. Indeed, if the aggregate values are computed using the k-values indicated in Table 3, priority vector I becomes optimal. Thus, using the decision theoretic technique in case where both the value score and preference weight are given in ordinal terms would be suspect.

5. CONCLUDING REMARKS

We have presented methodologies based on multi-attribute utility theory (MAUT) and fuzzy set analysis for doing decision analysis imprecisely but rigorously. Although there may be imprecision in the inputs, precise decisions could be made in both methodologies. The main focus of the article was to apply these methodologies to the problem of selecting a hazardous waste disposal

Table 8. Decision Matrix

		Priority		
		1	*2*	*3*
Alternative	1	$k_2+2k_5+k_{10}$	$k_1+k_3+k_4+k_5$ $+k_6+k_7+k_8+k_9$	$2k_1+k_2+2k_3+2k_4$ $+2k_6+2k_7+2k_8$ $+2k_9+k_{10}$
	2	$k_1+k_5+k_6+k_7$ $+k_8+k_9$	$k_2+k_3+k_4+k_5$	$k_1+2k_2+2k_3+2k_4$ $+k_5+k_6+k_7+k_8$ $+k_9+2k_{10}$
	3	$2k_1+2k_2+k_4$ $+2k_7+2k_8+2k_9$ $+2k_{10}$	$k_1+k_2+k_3+k_5$ $+k_6+k_7+k_8+k_9$ $+k_{10}$	$2k_3+k_4+2k_5$ $+2k_6$

Table 9. Possible Priorities Among Alternatives

Possibility	*Priority Vector*	*Aggregate Value*
I	[1 2 3]	$2k_2+3k_3+2k_4+4k_5+2k_6$ $+k_{10}$
II	[1 3 2]	$2k_1+4k_2+3k_3+2k_4+4k_5$ $+2k_6+2k_7+2k_8+2k_9+4k_{10}$
III	[2 1 3]	$2k_1+3k_3+2k_4+4k_5$ $+4k_6+2k_7+2k_8+2k_9$
IV	[2 3 1]	$4k_1+4k_2+3k_3+4k_4+2k_5$ $+2k_6+4k_7+4k_8+4k_9+4k_{10}$
V	[3 1 2]	$4k_1+2k_2+3k_3+2k_4+k_5$ $+4k_6+4k_7+4k_8+4k_9+2k_{10}$
VI	[3 2 1]	$4k_1+4k_2+3k_3+4k_4+k_5$ $+2k_6+4k_7+4k_8+4k_9+3k_{10}$

site where the alternatives were evaluated on the basis of multiple criteria (or objectives). In the particular case study examined, plant A was chosen as the optimal site under almost all conditions of impreciseness.

The relative merits of the two approaches are also examined. In the case study, the MAUT approach proved superior to the fuzzy set approach in the cases where the imprecision was either in the value score or in the preference weights (scaling constants) but not both. Both methodologies gave the same result but the MAUT based approaches required no more than ordinal information on the imprecise parameters. On the other hand, in the case where both preference weights and value scores are known imprecisely, the MAUT approach results in a clear decision which is suspect considering the many other cases studied. Thus, we would recommend the fuzzy set approach for the case of complete impreciseness. In such a case, the decision maker would have to provide more precise information than ordering among the parameters.

We would argue that the results from our case study could be generalized: the MAUT based approach would be most suitable when the decision maker responds with ordinal information on either the value scores or preference weights but not both. If the decision maker can only provide ordinal information on both, the fuzzy set approach is most suitable. As Watson et al., have pointed out [20], the danger in using fuzzy decision analysis is that the methodology is not that transparent to decision makers. In addition, the foundations of fuzzy-set theory are yet to be based firmly on philosophical and psychological grounds. Thus, while we agree with the conclusions of Watson et al. that "there are some theoretical reasons for preferring the fuzzy to the probabilistic" [20], and would add from our experience that it would be advantageous to use fuzzy set methodologies in cases of complete impreciseness, we would hesitate to recommend it strongly even in such a case.

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Direct reprint requests to:

G. Anandalingam
Department of Systems
University of Pennsylvania
Philadelphia, PA 19104