

A MATHEMATICAL MODEL FOR THE ACTIVATED SLUDGE PROCESS AT THE DETROIT WASTEWATER TREATMENT PLANT

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ABSTRACT

A model based on the balance of mass conversion rate is developed to simulate the function of aeration tanks at the Detroit Wastewater Treatment Plant. Runge Kutta fourth order numerical analysis is employed to compute effluent soluble BOD and mix-liquor VSS with designed experimental parameters. A fairly good agreement with low standard error of estimates between the calculated and experimental data is obtained. The model is able to not only predict at least forty-eight hours period of BOD and VSS; but also explain several features in activated sludge process such as return sludge, shock loading, steady state, and more importantly its dynamic nature.

One of the objectives in the biological treatment of wastewater is removal and coagulation of the soluble and nonsettleable colloidal organic material. The activated sludge process has been employed extensively for this purpose in wastewater treatment. Theoretical aspects of the process including model [1, 2], steady state kinetics [3, 4], computer simulation [5, 6], and optimization [7, 8] have been studied and reviewed [9]. However, it seems that a direct and overall comparison between the treatment plant data from the secondary system and the theoretical ones has been lacking. It is the objective of this article to correlate the actual data and the model.

Since the characteristics of wastewater vary dramatically from plant to plant rendering different removal results, the parameters in a theoretical model would have to be adjusted in accordance with the actual data collected under various experimental conditions. Indeed, only in this manner can it provide a reliable test. Here we have applied the Thomas method [10] to adjust the parameter γ proposed by Teissier [11], to more accurately describe a mixed culture system. A time lag between the influent to the aeration tank and return sludge, which is often overlooked has been incorporated into the model. As a consequence, the data generated by the model are more compatible with the field results. This empirical model will act as an aid to the basic understanding of the activated sludge process in this plant and will serve as a guide to process optimization.

EXPERIMENTAL

The experiment was designed to take flow readings and to collect samples *simultaneously* at ports A, B, C, D, and E in Figure 1. Samples were used for determining soluble BOD₅, S and volatile suspended solids, VSS. For five consecutive days including a week-end (when the nature of wastewater would be different), samples were taken every three hours, twenty-four hours a day. Then, the BOD and VSS were determined for each sample collected. Since the model designated ports B, C, and D with the same concentration of BOD, S is the average value of these three ports.

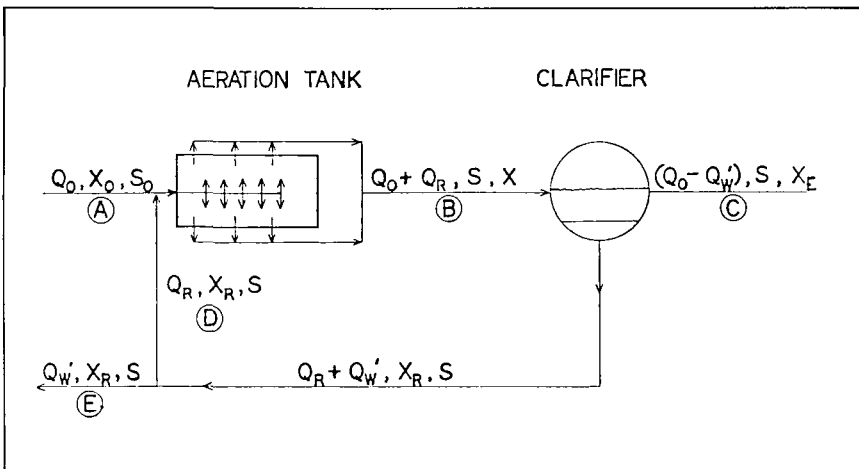


Figure 1. The Secondary system of a complete-mix aeration reactor

THE MODEL

A Complete-Mix Aeration Tank with Cellular Recycle —Waste from Sludge Line (in contrast to sludge wasting from mix-liquor line)

Since the secondary treatment process occurs primarily in the aeration tank, the reaction kinetics has to be defined in this tank. Therefore, the instantaneous reaction rates will be formulated *only* in the aeration tank. We will treat, however, final clarifiers separately later in conjunction with the optimized aeration tank. The problem will become simpler, then, since only the mass balance requirement is needed for optimizing the secondary clarifier unit operations.

The two time-dependent function $X(t)$ and $S(t)$ are given below. Considering the rate change of mass conversion, we have the simultaneous first order differential equation (1) and (2):

$$\frac{VdX(t)}{dt} = \left(\frac{YdS_B(t)}{dt} - k_d X(t) \right) V + Q_o X_o + Q_r X_r(t + \Delta t) - (Q_o + Q_r) X(t) \quad (1)$$

$$\frac{VdS(t)}{dt} = Q_o S_o + Q_r S(t + \Delta t) - (Q_o + Q_r) S(t) - \frac{VdS_B(t)}{dt} \quad (2)$$

The established equation [12] for bacterial growth is

$$\begin{aligned} \frac{dS_B}{dt} &= \frac{1}{\frac{dX}{dS_B}} \frac{dX}{dt} = \frac{1}{Y} (\mu X) \\ &= \frac{Y}{X} \left(m K_{T_o} (1 - e^{-\gamma S}) \right) \text{ following the Teissier equation [11]} \\ &= \frac{X}{Y} m K_{T_o} \theta^{(T-T_o)} (1 - e^{-\gamma S}), \end{aligned} \quad (3)$$

since Teisser equation takes temperature effect and food efficiency into consideration and these parameters can be adjusted for various systems. It is more convenient to incorporate these parameters into our formulas for later adjustment.

From Taylor's formula, $X_r(t + \Delta t)$ can be expanded into:

$$X_r(t + \Delta t) = X_r(t) + \frac{dX_r(t)}{dt} \Delta t + \dots$$

$$\cong X_r(t) + \frac{dX_r(t)}{dt} \Delta t \text{ by taking first-order approximation;} \quad (4)$$

and from mass balance,

$$X_r(Q_r + Q'_w) + X_e(Q_o - Q'_w) = X(Q_o + Q_r)$$

$$X_r = \frac{X(Q_o + Q_r) - X_e(Q_o - Q'_w)}{Q_r + Q'_w}.$$

In a system with a properly operating settling unit, the quantity of cells in the effluent is very small, i.e., $X_e = 0$; and under normal operation, the flowrate of each section would not be changed appreciably. Therefore,

$$\frac{dX_r}{dt} = \frac{Q_o + Q_r}{Q'_w + Q_r} \frac{dX}{dt} \quad (5)$$

Substituting equations (3), (4), and (5) into equation (1) and simplifying; one obtains equation (6)

$$\frac{dX(t)}{dt} = \frac{1}{1 - \frac{Q_r(Q_o + Q_r) \Delta t}{V(Q'_w + Q_r)}} X(t) \left(m K_{T_o} \theta^{(T-T_o)} (1 - e^{-\gamma S(t)}) \right) \quad (6)$$

$$-k_d - \frac{Q_o + Q_r}{V} + \frac{Q_o X_o + Q_r X_r(t)}{V}$$

In a similar way, equation (2) can be transformed into equation (7):

$$\frac{dS(t)}{dt} = \frac{V}{V - Q_r \Delta t} \left(\frac{Q_o}{V} (S_o - S(t)) \right) - \frac{X}{Y m} K_{T_o} \theta^{(T-T_o)} (1 - e^{-\gamma S(t)}) \quad (7)$$

Simultaneous differential equations (6) and (7) are the final expressions of our model. The approximations neglect higher-order terms than first order. One can, however, include higher-order terms by adding more first order differential equations, e.g., if a second order term is considered, one first order differential equation would be needed as required by Runge Kutta (RK) numerical analysis.

Variables

Independent:

t = reaction time in hours in the secondary process

Dependent:

$X(t)$ = concentration of microorganisms or volatile suspended solids, VSS, mg/l.

$S(t)$ = concentration of waste, substrate, or food surrounding the microorganisms, soluble BOD, mg/l.

$S_B(t)$ = concentration of substrate in aeration tank, mg/l.

Parameters

Q_o = flow rate of primary effluent to aeration tank (PE to AS) without return sludge, million liters/day, MLD.

Q'_w = flow rate of return sludge, MLD.

Q_r = flow rate of waste sludge, MLD.

X_o = VSS concentration of PE to AS, mg/l.

X_e = VSS concentration of final effluent, mg/l.

X_r = VSS concentration of return sludge, mg/l.

S_o = substrate concentration of PE to AS, mg/l.

V = volume of the aeration tank = 67.4 ML.

T = temperature of the activated sludge in the aeration tank.

Δt = time lag between aeration tank effluent and return sludge influent to the aeration tank, hours.

Constants

θ = temperature activity coefficient = 1.0 – 1.03 for AS [13].

γ = a constant from Teissier equation [11] = 0.0125/mg/l.

$m^{K_{T_o}}$ = maximum specific food utilization rate at temperature T_o or maximum rate of waste utilization/unit weight of microorganisms = 3.6/day at $T_o = 20^\circ\text{C}$ [14].

k_d = microorganism decay coefficient [15] = 0.06/day.

Y = growth-yield coefficient [16], mass of microorganism/mass of substrate utilized = 0.6.

RESULTS AND DISCUSSION

(A) Application of the Thomas Technique to Attaining Plant Parameters [10]

Numerical solutions of simultaneous non-linear differential equation (6) and (7) were obtained by using the RK fourth order analysis with a CDC computer. All data required were collected from the actual operation. Before the Thomas method was applied to our data set, all the steady state values of X and S were calculated by inserting the available data into equations (6) and (7) where dX/dt and dS/dt were set to zero. It is of no surprise that the majority of experimental X and S values are, in fact, the steady state values as are shown in Table 1. This reflects the fitness of the model of the plant data in the first

Table 1. Comparison Between Experimental and Calculated Steady State VSS and BOD

<i>t, hr</i>	<i>X, mg/l</i>	<i>S, mg/l</i>	<i>X, mg/l</i>	<i>S, mg/l</i>
3	1572	5	1689	5
7.5	1628	6	1499	6
9	1076	5	1099	5
12	1564	5	1523	5
15	1604	4	1616	4
18	1500	5	1367	5
27	1544	4	1457	4
30	1528	4	1509	4
33	1404	4	1391	4
35	1380	4	1394	4
39	1500	3	1457	3
42	1496	3	1406	3
45	1664	3	1516	3
54	1468	3	1385	3
57	1552	3	1521	3
60	1564	3	1640	3
63	1628	3	1587	3
69	1424	3	1341	3
73	1512	3	1438	3
85	1500	3	1397	3
89	1404	3	1292	3
97	1292	2	1224	2
109	1308	3	1226	3
113	1320	2	1376	2
117	1248	3	1186	3

approximation. The expression used for calculating these steady state values was the common solution for equation (6) and (7):

$$X = \frac{1}{k_d V + Q_o + Q_r} (Y Q_o (S_o - S) + Q_o X_o + Q_r X_r) \quad (8)$$

where only constants k_d and Y were taken from the literature [15, 16]. It was unnecessary to know γ_m , K_{T_o} , and θ . Indeed, these constants can be obtained from the Thomas method.

It was implied in the Thomas scheme that any equation of the form

$$Y = L(1 - 10^{-kt}) \quad (9)$$

can be transformed into the format:

$$\left(\frac{t}{y}\right)^{1/3} = (2.3 kL)^{-1/3} + \frac{k^{2/3} t}{3.43 L^{1/3}} \quad (10)$$

by applying simple algebra. Using the linear relationship of $(t/y)^{1/3}$ vs. t in equation (10), one can obtain constants K and L which originally represented the decay constant and ultimate BOD in a biological culture. The one-to-one correspondence of the symbols in equations (6) and (7) to Thomas' terminology is as follows:

$$\begin{aligned} K &\rightarrow \gamma/2.3 \\ y &\rightarrow \frac{1}{X} \frac{Q_o(S_o - S)}{V} \text{ or } k_d + \frac{Q_o + Q_r}{V} - \frac{Q_o X_o + Q_r X_r}{XV} \\ t &\rightarrow S \\ L &\rightarrow \frac{m K_{T_o}}{Y} \theta(T - T_o) \end{aligned}$$

Using the calculated steady state values in Table 1, the plots of

$$\left(S / \left(k_d + \frac{Q_o + Q_r}{V} - \frac{Q_o X_o + Q_r X_r}{XV} \right) \right)^{1/3} \text{ vs. } S \text{ and } (SXV/Q_o(S_o - S))^{1/3} \text{ vs. } S$$

are shown in Figure 2. From the intercepts, γ was determined to be 0.12/(mg/l) and 0.26/(mg/l) as compared to 0.0125/(mg/l) obtained by Teissier [11]. However, unreasonably large values 8.5 – 11.0/(mg/l) were derived from the slopes. It should be pointed out that the inconsistency probably arises from the fact that equation (10) is an approximation from the expansion of

$$L/Y = e^{2.3kt} / (e^{2.3kt} - 1) \cong \frac{1 + 2.3kt + \frac{(2.3kt)^2}{2!} + \frac{(2.3kt)^3}{3!}}{(1 + 2.3kt) - 1}$$

Additionally, the value 3.34 in the second term of equation (10) can never fit perfectly the last three terms of the third power on the right-hand side of the equation. Thus, the γ value was tested substantially within the range obtained. It appears that the best value in RK numerical analysis is 0.0125 – 0.12 for the steady state and 0.26/(mg/l) for transient cases (see following section) implying that $m K_{T_o}/Y = 6.0$. Hence, the set $Y = 0.6$ and $m K_{T_o} = 3.6/\text{day}$ appear to be reasonable values consistent with those reported in the literature [14, 16].

(B) Dynamic Behavior of the Activated Sludge Process at DWTP

Perhaps, the best challenge to the model is to see if it describes the dynamic behavior of actual plant data as the activated sludge process continues. The idea here was to introduce all of the field data except S and X into equations (6) and (7) while RK numerical analysis was being carried out. Naturally, the initial conditions for this numerical computation were at $t = 0$; namely, the time when

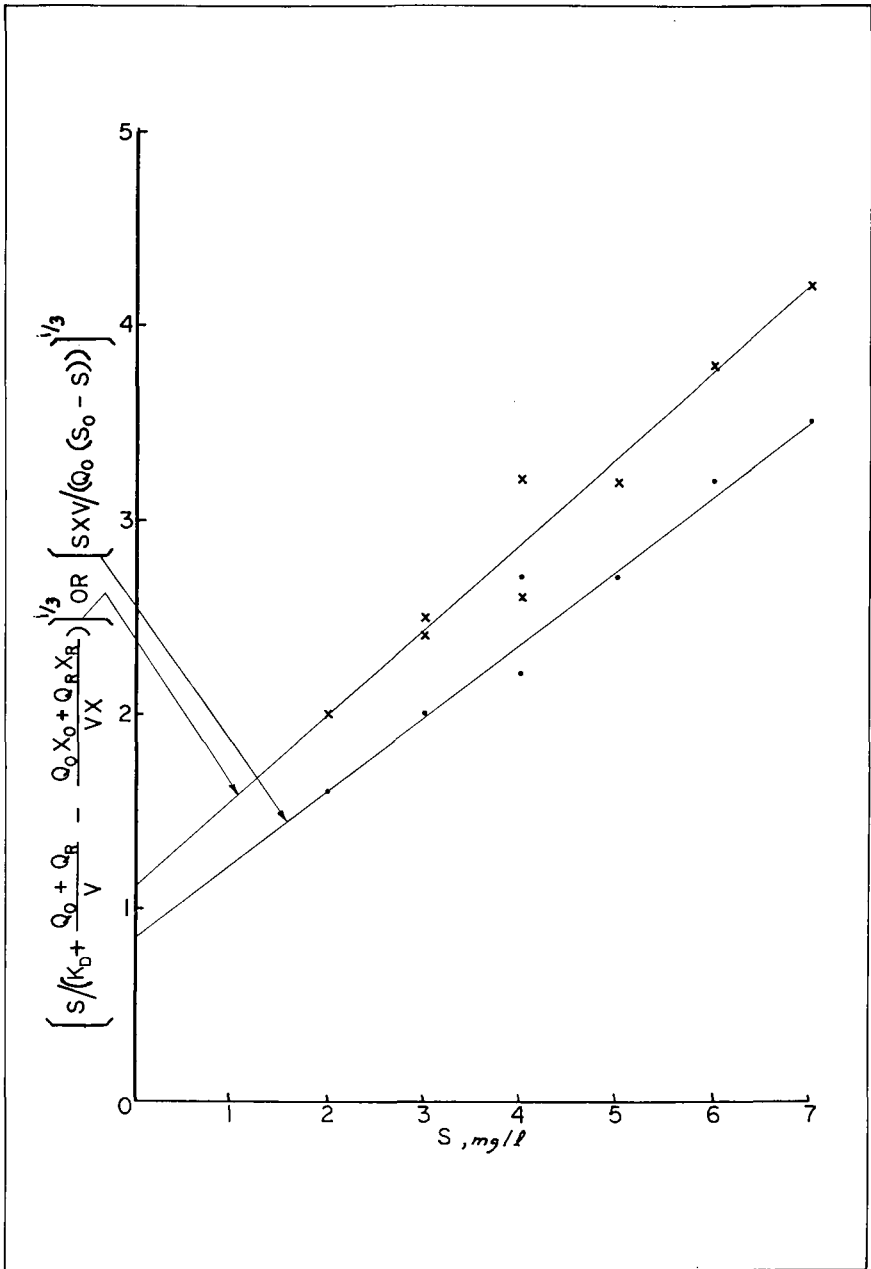


Figure 2. Determination of a Parameter γ from the Model and with Thomas Scheme.

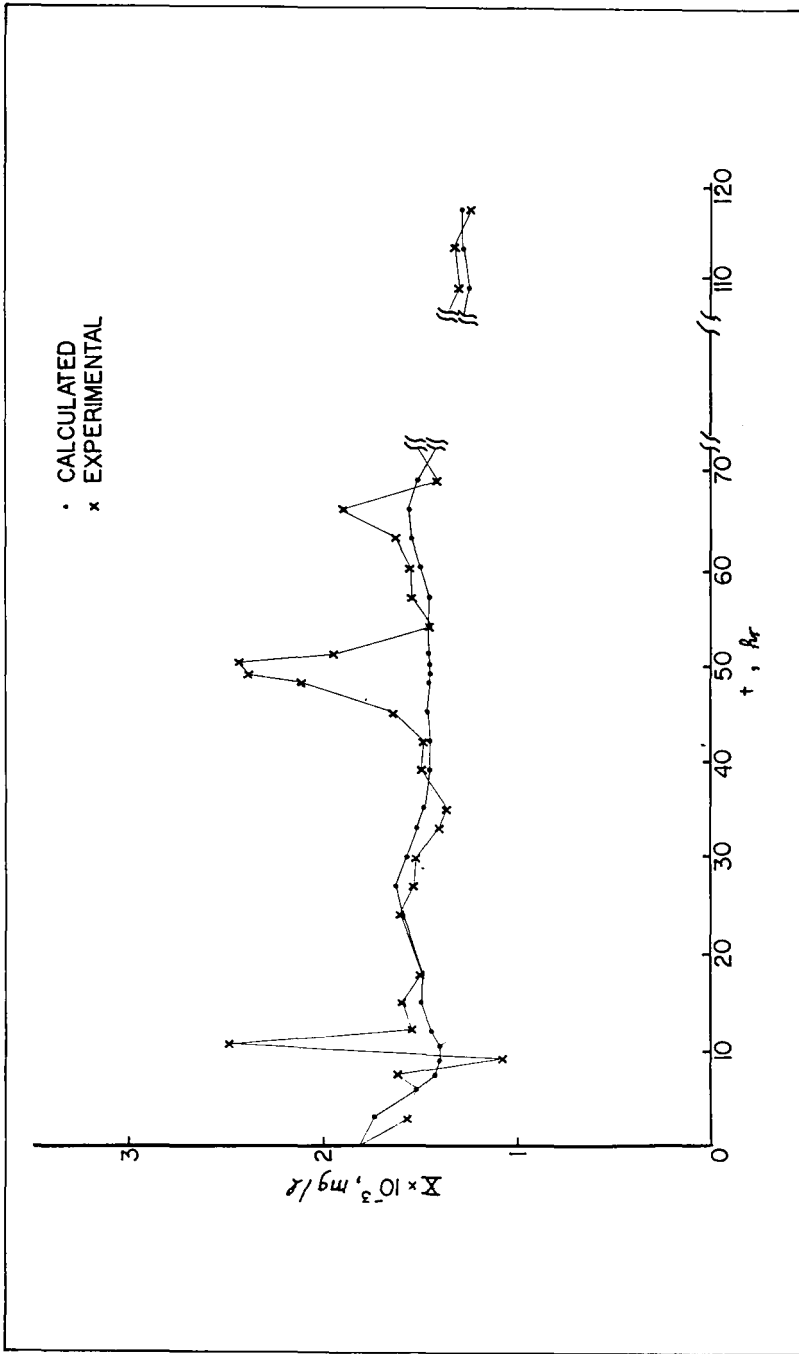


Figure 3. A comparison of calculated and experimental VSS data.

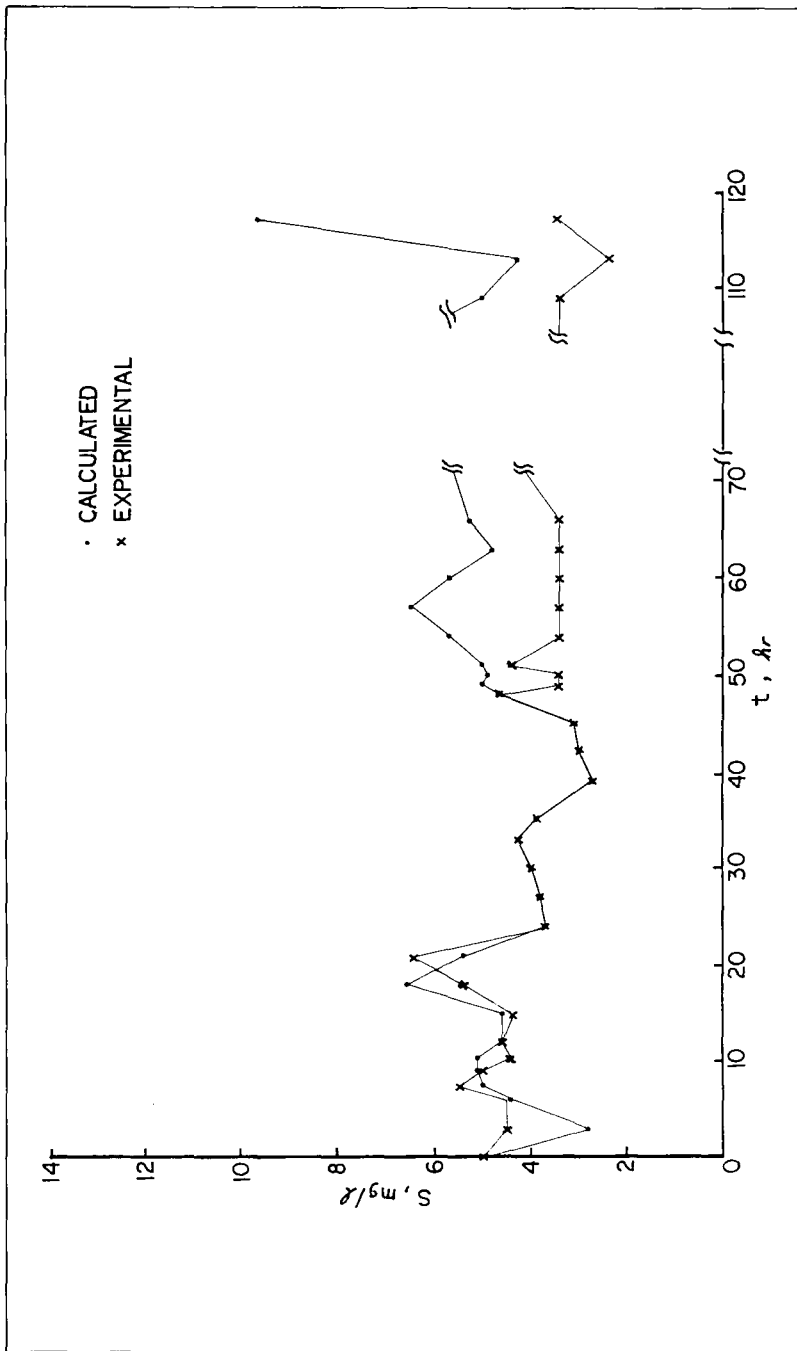


Figure 4. A comparison of calculated and experimental BOD data.

the first set of samples was collected. In order to reduce instabilities (which often occur in numerical analysis) due to sudden changes of the parameters Q_o , Q'_w , Q_r , X_o , X_r , and S_o , the linear relationship between consecutive values rather than step functions was taken. This assumption is closest to the natural condition as long as the time span between consecutive data items is not too large. The program was written in such a way as to read in the experimental data sequentially and print out predicted values of VSS and BOD at the same time interval for comparison.

Both Figures 3 and 4 display the comparison between calculated and experimental data for VSS and BOD, respectively. Constant, γ was taken as 0.26/(mg/l) in these computations. The transient nature of the BOD and VSS was revealed clearly and completely by the model. It fits almost exactly for the first two days of BOD data and fairly well for the overall VSS data which vary significantly. It would have been extremely difficult to obtain the correct order of magnitude for the data had the model been unsuitable.

Two-variable regression analyses were carried out for Figures 3 and 4. The experimental set was selected as the independent variable; whereas the predicted set was taken as a dependent variable. The standard error of estimate, S_e , which measures the spread of given points around the fitted line of regression and the correlation coefficient, r (which is independent of the choice of variable) were computed. $S_e(S) = 0.62$ and $r(S) = 0.7995$ for BOD data within the first two days, whereas $S_e(X) = 118.56$ and $r(X) = 0.5101$ for all VSS data except five doubtful field values. A relatively larger deviation for BOD data after two days might be attributed to the accumulation of error in the repeated numerical integration. However, the predicted values are still in the same order of magnitude as experimental ones.

This empirical model with obtained constants is now being tested at DWTP for daily operation. Specifically, to predict the recovery period required and the values of BOD (soluble and total) and VSS after a shut-down of one unit operation in the chain process of the secondary system. It is felt that a dispersion factor [17] which considers the admixture of plug-flow characteristics in a complete-mix model should be researched. The incorporation of secondary clarifiers, too, ought to be regarded as an integral part of optimizing procedures of the secondary system. Both of these concerns are the objects of future study.

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