The influence of refraction on wind turbine noise

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A semi-empirical method is employed to calculate the time-average sound level of the wind turbine noise generation and propagation. Both are affected by refraction due to wind speed gradient. Under upwind conditions the partially ensonified zone separates the fully ensonified zone (close to the turbine) and the shadow zone (far away from the turbine). Refraction is described in terms of the wind speed linear profile fitted to the power law profile. The rotating blades are treated as a two-dimensional circular source in the vertical plane. Inside the partially ensonified zone the effective A-weighted sound power decreases when the receiver moves from the turbine toward the shadow zone. It is assumed that no sound energy is diffracted and scattered into the shadow zone.

1. INTRODUCTION
A large increase in wind power is mandated all over the world. However, wind turbine noise appears to be a rather disturbing sound when compared to other noise sources [1]. In many countries the noise limits of wind turbine noise are expressed in terms of the time average sound level, $L_{AeqT}$[2]. Unfortunately, even distant turbine noise evokes annoyance, so refraction can’t be ignored. Fig.1 illustrates the influence of wind speed gradient on sound propagation.

As expected, for downwind propagation the value of $L_{AeqT}^{(down)}$exceeds the upwind value of $L_{AeqT}^{(up)}$at the same horizontal distance from the turbine (Fig.2, see Ref.[3]). The purpose of this study is to find the dependence of $L_{AeqT}$ on the horizontal distance from the turbine tower.

For a point source at the height $z$ (Fig.3) the limiting ray grazes the ground at the horizontal distance $x = \rho$ and produces a shadow boundary at (the shaded area). The distance to the grazing point, $\rho$, is a function of the source height, $z$ [4,5]. Note that inequality $x < \rho$ determines the fully ensonified zone (red line on Fig.3).

When the horizontal distance is equal to the tower height, $\rho = h$, only a segment of the rotor plane rim (between 2 and 4 o’clock) emits the sound (Fig.20 in Ref [6]). This is caused by the directivity of the trailing edge and the Doppler amplification. In this study we consider the much larger horizontal distance, $\rho > 3h = 300 m$ where both
effects, (directivity and amplification) are negligible. Under such circumstances, the bulk of the sound power is uniformly distributed along the rim of the rotor plane in the $y-z$ plane (Fig.4b). Thus the wind turbine can be modeled by points sources located along the circle of radius $l$, whose heights $z$ increase from $h-1$ to $h+1$. Each point of a this circle radiates its own limiting ray (Fig.3). Consequently, there are limiting rays for the lowest and highest point of the rotor plane, which graze the ground at the horizontal distances $r_1$ and $r_2$, respectively (Fig.4a). For these two points the fully insonified zones and shadow zones are defined by the inequalities $x < \rho_1; x < \rho_2$ and $x < \rho_1; x < \rho_2$, respectively. Finally, two inequalities

$$\rho_1 < \rho < \rho_2$$  \hspace{1cm} (1)$$

describe the location of partially insonified zone [7]. This zone is between the fully ensonified zone, $x < \rho_1$ (red line), and the shadow zone, $x < \rho_2$ (navy blue line). Neglecting diffracted and scattered waves, which penetrate into the shadow zone, for $x < \rho_2$ one could expect complete silence [5]. Within the partially insonified zone (Eq.1), the time average sound level, $L_{AeqT}$, decreases from $L_{AeqT(\rho_1)}$ to $L_{AeqT(\rho_2)}$. In reality this decrease equals about 20-30 decibels. In Sect.5 it will be shown that the receiver translation from $\rho_1$ to $\rho_2$ is accompanied by switching off some point sources on the rotor rim (Fig.4b).

Figure 2: Time variations of the A-weighted sound pressure level, $L_{pA}$, at the horizontal distance 1000 m, for down- and upwind propagation (see Fig.1 and Ref.[3]).

Figure 3: For a point source above the ground, the limiting ray grazes the ground at the horizontal distance $x = \rho$ and makes the shadow zone at $x > \rho$. 
The generation of wind turbine noise has been investigated in both experimental and theoretical studies (see Ref. [6] and the literature cited therein). There are empirical relationships between the A-weighted sound power level and the blade tip speed, rotational speed, and hub wind speed. The semi-empirical model presented in Sect. 3 employs these relationships.

2. REFRACTION

Refraction is quite a well-known phenomenon. The limiting rays graze the ground and make a shadow boundary in the vertical plane (Fig.5). To determine the distance to the shadow zone, we apply the effective sound speed in a stratified atmosphere [4,8,9],

$$c(z) = c_{ad}(z) - V(z) \cos \alpha$$  \hspace{1cm} (2)

In the above formula, $c_{ad}$ is the adiabatic speed of sound which depends on the air temperature and the product $V(z) \cos \alpha$ denotes the wind speed component in the direction of sound propagation from the turbine tower to receiver (Fig.5). Here $\alpha$ is the angle between the normal to the rotor plane (x axis) and the ray direction in the x-z plane (bird’s eye view). The variation in wind speed with the height $z$ is usually described by

$$V(z) = V(h) \left( \frac{z}{h} \right)^{\beta}$$  \hspace{1cm} (3)

where $V(h)$ is the wind speed at the hub height $h$ (Fig.4). Two wind speed...
measurements at two heights, \( V(z_1) \) and \( V(z_2) \), yield the wind shear exponent (Eq.3). This exponent is a function of surface roughness and air stability. In Ref. [10] one can find that for urban and rural areas \( 0.15 < \beta < 0.3 \) and \( 0.07 < \beta < 0.55 \), respectively.

To obtain the final equations as simply as possible, we fit a linear profile,

\[
V(z) = V(h) \left[ a \frac{z}{h} + b \right]
\]  

(4)

to the power law wind profile (Eq.3). The unknown parameters \( a \) and \( b \) minimize the average error,

\[
\sigma^2(a,b) = \frac{1}{h} \int [V(z) - V(z)]^2 dz
\]  

(5)

and the conditions,

\[
\frac{\partial \sigma^2}{\partial a} = 0
\]  

lead to

\[
a = \frac{6\beta}{(1+\beta)(2+\beta)} \quad b = \frac{2(1-\beta)}{(1+\beta)(2+\beta)}
\]  

(6)

We assume that the adiabatic component of the vertical gradient, \( dc_{ad}/dz \), is relatively small as compared with the vertical gradient of the wind speed component (Eq.4),

\[
\frac{dV}{dz} \approx a \cdot \frac{V(h)}{h}
\]  

(8)

Consequently, the linear approximation of the effective sound speed in a stratified atmosphere (Eq.2) takes the form,

\[
c(z) = c_{ad}(0) - \frac{6\beta}{(1+\beta)(2+\beta)} \frac{V(h) \cos \alpha}{h} \cdot z
\]  

(9)

Here \( c_{ad}(0) \) expresses the adiabatic speed at ground level (e.g. \( c_{ad}(0) = 340m/s \) for the air temperature of 15°C). Finally, for a point source at the height \( z \), the horizontal distance between the turbine tower and the shadow boundary at the ground, \( \rho \) (Fig.5) can be calculated from [4,5],

\[
\rho(\alpha) = \mu \frac{zh}{\cos \alpha}
\]  

(10)

where

\[
\mu = \sqrt{\frac{(1+\beta)(2+\beta) c_{ad}(0)}{3\beta}} \frac{V(h)}{h}
\]  

(11)

If the receiver moves along the x axis, then \( \alpha=0 \) and \( \rho(0)=x \) (Fig.5). Accordingly, the location of the partially insonified zone on the \( \chi \) axis is determined by (Fig.4, Eq.10),

\[
x_1 = \rho_1 = \mu \sqrt{h \cdot (h-l)}
\]  

(12)

\[
x_2 = \rho_2 = \mu \sqrt{h \cdot (h+l)}
\]  

EXAMPLE 1

For weakly stable air conditions in an urban area, \( \beta = 0.3 \). Then the adiabatic speed close to the ground surface \( c_{ad}(0) = 340m/s \) and the hub height wind speed \( \zeta(\eta) = 15m/s \) give \( \nu = 8.67 \) (Eq.11). If the turbine height \( h = 100m \) and blade length \( l = 40m \), then the transition zone is determined by and \( \rho_1 = 670m \) and \( \rho_2 = 1025m \) (Fig.4).

3. NOISE GENERATION

The airfoil self-noise and inflow-turbulence noise prevail over the mechanical noise, blade-tower interaction noise, cooling fan noise, and eagle impact noise. The results in [7] make it possible to write the A-weighted sound power level of wind turbine as a logarithmic function of the blade tip speed, \( \nu \) (Fig.6),

\[
L_{wA} = 10 \log \frac{\nu}{\nu_0} + A \quad \nu_0 = 1m/s
\]  

(13)

For the blade length of \( l \) [m] and rotational speed \( N \) [rps],

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(14)

\[ v = 2\pi N \cdot l \]

where 2<n<5. The blade tip sweeps through a wind that changes with the height above ground, z. The A-weighted sound power of sound from a single tip is proportional to the n-th power of the wind speed at the momentary height, \( V(z) \),

(19)

\[ w \propto \left( \frac{V(z)}{v_o} \right)^n \quad h-l < z < h+l \]

(15)

\[ L_{WA} = 10m \cdot \log \frac{N}{N_o} + A \quad N_o = 1rps \]

In Refs.[11-13] one can find the measurement results that are characterized by 4<\( \mu \)<7.

The definition of the A-weighted sound power level is

(16)

\[ L_{WA} = 10 \log \frac{W_A}{W_o} \quad W_o = 10^{-12} \]

where \( W_A \) expresses the A-weighted sound power. Ultimately, the empirical results given by Eq.(13) and (15) lead to the conclusion that the A-weighted sound power from a blade tip is proportional to the m-th power of its speed \( v \),

(17)

\[ w \propto \left( \frac{v}{v_o} \right)^m \quad v_o = 1m/s \]

On the other hand, measurements [14-16] show that the A-weighted sound power level of turbine noise, \( L_{WA} \), and the inflow wind speed at the hub height, \( V(h) \), are related to each other by,

(18)

\[ L_{WA} = 10n \cdot \log \frac{V(h)}{v_o} + B \]

where \( 2<n<5 \).

Figure 6: The blade tip moves along the circular trajectory at the distance \( l \) from the hub with the speed (Eq.14).
Here the time varying phase, $\Phi = 2\pi \Omega t$, where $\Omega [rps]$ denotes rotational speed.

For both categories of airfoil self-noise and inflow-turbulence noise, the A-weighted sound power due to a single blade takes the form (Eqs.17,20),

$$ w(t) = q \left( \frac{v}{v_e} \right)^m \left( \frac{V(h)}{v_e} \right)^n F(t) $$

where $q$, as well as $m$ and $n$ denote the free parameters which could be found from $L_{WA}$, $v$, the $V(h)$ and measurements (see below). Mindful of the time period, $T = 1/\Omega$, the time average value of the A-weighted sound power from a single blade can be written as (Eqs.22, 23),

$$ W_A = q \left( \frac{v}{v_e} \right)^m \left( \frac{V(h)}{v_e} \right)^n \langle F \rangle $$

with the factor

$$ \langle F \rangle = 1 + \frac{1}{4} n\beta(n\beta - 1) \left( \frac{h}{l} \right)^2 $$

The above expression describes the effect of wind shear, $\beta$, on wind turbine sound power: for $0 < n\beta < 0.5$ the value of $W_A$ decreases with $\beta$. Then, for $n\beta > 0.5$ the value of $W_A$ grows. The electric wind turbine power exhibits similar behavior [17].

4. FULLY ENSONIFIED ZONE

Suppose the downwind receiver (Figs.1,3) moves close to the ground ($z \rightarrow 0$) from the wind tower along the line of constant angle $\alpha$ (Fig.5). The receiver gets the sound energy from all the point sources of the rotor rim (Fig.7a), when its horizontal distance to the tower, $\rho(\alpha)$, is less than (Eq.10 $\varepsilon$ with replaced by $h-l$),

$$ \rho(\alpha) = \mu \frac{(h-l)z}{\cos \alpha} $$

In other words, inside of the fully ensonified zone, $\rho(\alpha) < p_1(\alpha)$, all point sources of the rotor rim participate in the noise at the receiver. If the wind shear effect is small ($n\beta << 1$ and $F = 1$ see Eq.25), the formula (24) simplifies to the form

$$ W_A = q \left( \frac{v}{v_e} \right)^m \left( \frac{V(h)}{v_e} \right)^n $$

Then we take into account three blades

![Figure 7](image-url)

**Figure 7:** Within fully insonified zone, $0 < \rho < \rho_1(\alpha)$ (Eq.26), all point sources of the rotor rim (figure a) contribute to noise at the receiver. Within transition zone, $\rho_1(\alpha) < \rho_2(\alpha)$ (Eq.30), only percentage of point sources of the rim, $0 < \alpha < \pi/2$ (figure b), supply sound energy to the receiver. Within the shadow zone, $\rho > \rho_2(\alpha)$ (Eq.30), no point source of the rim (figure c) contributes to noise at the receiver.
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and substitute $3W_A$ into the definition of the A-weighted sound power level (Eq.16). Thus we arrive at the semi-empirical equation,

$$L_{WA} = 10m \cdot \log \frac{v}{v_o} + 10n \cdot \log \frac{V(h)}{v_o} + C \quad (28)$$

The constant $C$, as well the parameters $m$ and $n$, can be determined from $L_{WA}$, $v$ and $V(h)$ measurements. The measurements reported in Refs. [11-16] confirm the dependence of $L_{WA}$ (Eq.28) on the blade tip speed, $v$ (Eq.13), and on the wind speed at the hub height, $V(h)$ (Eq.18).

The engineering prediction models of the time average sound level, $L_{AeqT}$, are based on non-directional sound divergence over a flat ground surface and air absorption (e.g. Refs.[18-23]). In reality the hub height $h$ and the horizontal distance to the receiver $\rho$ meet the conditions: $h<100m$ and $\rho>150m$. Thus, within 1dB error, for the fully insonified zone, $0<\rho(\alpha)<\rho_1(\alpha)$ (Eq.26), one can write:

$$L_{AeqT} = L_{WA} - 10 \log \left[ \frac{2\pi^2}{\rho^2} \right] - 0.005 \cdot \frac{\rho}{h} \quad (29)$$

$L_{WA}$ can be calculated from Eq. (28).

5. PARTIALLY INSONIFIED ZONE

The red and navy blue lines in Fig.8 described by the functions,

$$\rho_1(\alpha) = \mu \sqrt{\frac{(h-1)h}{\cos \alpha}}$$

$$\rho_2(\alpha) = \mu \sqrt{\frac{(h+1)h}{\cos \alpha}}$$

with $\mu$ calculated from Eq.(11), define the borders of the partially insonified zone. The sound energy reaching any point of this zone (on the x-y plane, i.e. ground surface),

$$\rho_1(\alpha) < \rho(\alpha) < \rho_2(\alpha) \quad (31)$$

is emitted by point sources on the upper part of the rotor rim (Fig.7b). Due to refraction (Fig.4) the point sources of the lower part are not active and don’t participate in noise immission within the partially ensonified zone. The sound power of the active point sources equals (Fig.7b),

$$\dot{W}_A = \frac{\Phi}{\pi} \cdot W_A \quad (32)$$

where $W_A$ (Eq.27) denotes the A-weighted sound power which characterizes the noise immission inside the fully ensonified zone (Fig.7a). To determine the angle $\phi$ note that (Fig.7b)
Writing, \( r(\alpha) = r \), \( r_1(\alpha) = r_1 \), and \( r_2(\alpha) = r_2 \), we find from Eqs. (30) and (31),
\[
\cos \varphi = \frac{z - h}{l - h} \quad (33)
\]

So combination of Eqs. (33) and (34) yields
\[
\varphi = \arccos \frac{2r^2 - r_1^2 - r_2^2}{r_2^2 - r_1^2} \quad (35)
\]

If the receiver approaches the fully insonified zone, \( r \to r_1 \) (Fig.8, Eq.30) and \( \phi \to \pi \) (Eq.35), and then Fig.(7b) transforms into Fig.(7a) because the non-active sources disappear. Accordingly Eq.(32) gives: \( \bar{W}_A \to W_A \).

When the receiver moves in the opposite direction to the shadow zone, \( r \to r_2 \) (Fig.8, Eq.30), formula (35) brings about, \( \phi \to 0 \), because all the point sources on the rotor rim become non-active (Fig.7c). Then Eq.(32) yields, \( \bar{W}_A \to 0 \) . In other words, for the horizontal distance \( r_2 \), the receiver falls into the shadow zone without any turbine sound (Fig.8). Finally, formula below
\[
L_{WA} = L_{WA} + \Delta L_{WA}(\rho) = 10 \log \left( \frac{2\rho^2}{\rho^2} \right) - 0.005 \cdot \frac{\rho}{l} \quad (36)
\]
gives the time average sound level, when the receiver is within the partially insonified zone, \( \rho_1 < \rho < \rho_2 \) (Eq.31). Here \( L_{WA} \) can be calculated from a semi-empirical equation (28). For \( \rho_1 < \rho < \rho_2 \) the correction to the A-weighted sound power level equals,
\[
\Delta L_{WA}(\rho) = 10 \log \left( \frac{1}{\pi} \arccos \frac{2r^2 - r_1^2 - r_2^2}{r_2^2 - r_1^2} \right) \quad (37)
\]
with horizontal distances \( \rho_1 \) and \( \rho_2 \) defined by Eq.(30).

EXAMPLE 2

For wind conditions as in Example 1, the transition zone is determined by \( \rho_1 = 670m \) and \( \rho_2 = 1250m \). The correction \( \Delta L_{WA}(\rho) \) , for the partially ensonified zone \( \rho < \rho_1 < \rho_2 \) (Eq.37) is plotted in Fig. 9.

5. CONCLUSIONS

The semi-empirical theory presented here is based on measurements of: \( L_{WA} \) - A-weighted sound power level, \( v \) - blade tip speed, and \( V(h) \)- wind speed at the hub height. Due to wind shear induced refraction, the point sources on the rotor rim both, \( do, \) and \( do \) not contribute to the sound energy at the receiver (Fig.7). When the receiver is located within the fully insonified zone, \( \rho < \rho_1 \) (Eq.26, Fig.8), formulae (28) and

Figure 9: The effect of refraction within the partially ensonified zone, \( \rho_1 = 670m \) and \( \rho_2 = 1250m \) (Eqs.30,31): correction \( \Delta L_{WA} \) (Eq.37) to the A-weighted sound power level, \( L_{WA} \) (Eq.28), as a function of the horizontal distance, \( 670 < \rho < 1250m \) (Fig.8).
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(29) can be used for calculating of the time-average sound level, $L_{AeqT}$. Further away from the turbine, within the partially ensonified zone (Eqs.30, 31, Fig.8), expressions (36) and (37) can be applied. Measurements indicate (Fig.2) that at the same horizontal distance from the turbine, the downwind value of $L_{AeqT}$ exceeds the upwind value of $L_{AeqT}$. To quantify this effect we make use of Eqs.(36) and (37),

$$L_{AeqT}^{\text{down}} - L_{AeqT}^{\text{up}} = -10 \log \left( \frac{1}{\pi \cos \theta} \right) \left( \frac{\rho^2 - \rho_i^2}{\rho_i^2 - \rho_1^2} \right)^2$$

(38)

The method presented here cannot be applied in hilly terrain or in the coastal area.

REFERENCES


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**NOISE COMPLAINTS MAKE PUBLICAN GIVE-UP**

A Weymouth pub manager claims he has been forced out because of constant noise complaints. Cristian and Beatrise Neagu were managers of the John Gregory pub in Weymouth for over two years. Mr Neagu said “I’m devastated and so fed up with the noise complaints that it has forced us to leave the pub and Weymouth. Even at my own farewell gig, I have cancelled live music because I know that it will be followed with constant complaints. I even got a music level limiter enforced by the council, which shuts off the power once music levels get too high. I like to run my business in peace but every weekend I get complaints. I can’t run it like this and am made to feel like a criminal.” Debby Rose, secretary of the Southill Preservation Society, said: “It’s a sad thing if they are being driven out by complaints as they’ve done a fantastic job with the pub.”

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**LANDLADY FINED £9,000 FOR IGNORING NOISE ABATEMENT ORDERS**

A woman has been fined £9,000 for failing to comply with several noise abatement notices served on Rendezvous Bar and Grill on High Road, Chadwell Heath, East London. Between November 2012 and May 2013, Redbridge and Havering councils ordered the bar to turn down music on four occasions but the notices were contravened. Joanne Wilson initially pleaded not guilty to failing to comply with the prohibition of playing loud amplified music at the restaurant. However she changed her plea to guilty on September 19 at Romford Magistrates’ Court. She was fined £2,000 for each of the noise abatement notices that she controvened, totalling £8,000, and was ordered to pay and extra £1,000 in costs.