Consideration of Fire Development
In an Enclosed Space

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ABSTRACT
The equations governing the temperature build up in a fire room are derived. Graphical results are presented which show the effect of parameters such as wall heat transfer rates, combustion air, characteristics of the wall material, etc., on the temperature variation in the room.

Introduction
Current research in Fire Safety Technology relies on the usual methodologies and procedures available to scientists and engineers. These include theoretical computations of the phenomena involved, the use of physical models and mathematical models, the use of full scale tests and, finally, on the raw observations made by firefighters and others during real fire experiences.

The behavior of a fire as it runs its course through ignition, buildup, and eventual burnout or extinguishment is of importance to investigators interested in devising fire protection systems involving detectors, alarms, sprinklers, stair pressurization and exhaust, and in other studies of the action of fires on structures of various kinds.

The rate of combustion, temperatures and pressures reached, and the quantities and kinds of gases produced during a fire are all significant to the damage produced.

This paper describes the development of a simple model which will be useful for the prediction of temperature and other characteristics during fires (hypothetical) for which fuel loading, ventilation, space configuration and other necessary characteristics and environmental features are assigned.
The project is one of a series in fire safety research conducted by the Center for Urban Environmental Studies of the Polytechnic Institute of New York.

Analysis

In the actual case of a fire in a room, the fire is initiated at one point in the room and then spreads to include the combustion of all flammable material. As the fire consumes more of the combustible material the pressure in the room increases slightly. In addition, the hot combustion gases rise causing a vortex type motion which brings cool air into the fire region. If there are windows in the room, they are normally shattered as the temperature in the room exceeds several hundred degrees Fahrenheit. This results in the venting of the hot combustion gases from the room and increased flow from the ambient into the room.

In the present analysis, a simplified model is assumed for the combustion of material in a room. (See Table 1 for list of symbols.) Figure 1 shows a schematic

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>defined by Equation 3</td>
</tr>
<tr>
<td>A</td>
<td>total surface area of room, m²</td>
</tr>
<tr>
<td>A_s</td>
<td>total solid surface area of room, m²</td>
</tr>
<tr>
<td>A_w</td>
<td>total window area of room, m²</td>
</tr>
<tr>
<td>b</td>
<td>defined by Equation 3</td>
</tr>
<tr>
<td>C</td>
<td>specific heat of wall material, kcal/kg°C</td>
</tr>
<tr>
<td>C_p</td>
<td>mean coefficient of specific heat at constant pressure of combustion gases, kcal/kg°C</td>
</tr>
<tr>
<td>G_o</td>
<td>volume of gases produced by the combustion of the fuel, m³/kg</td>
</tr>
<tr>
<td>H</td>
<td>effective heating value of fuel, kcal/kg</td>
</tr>
<tr>
<td>H_crg</td>
<td>enthalpy of combustion gases, kcal</td>
</tr>
<tr>
<td>H_f</td>
<td>enthalpy of gas mixture in room, kcal</td>
</tr>
<tr>
<td>H_comb</td>
<td>rate of energy supplied by the combustion of fuel, kcal/hr</td>
</tr>
<tr>
<td>H_f</td>
<td>rate of change of enthalpy of gases in the fire room, kcal/hr</td>
</tr>
<tr>
<td>H_m_a</td>
<td>enthalpy rate at which air enters fire room, kcal/hr</td>
</tr>
<tr>
<td>H_m_o</td>
<td>enthalpy rate at which combustible gases leave room, kcal/hr</td>
</tr>
<tr>
<td>I</td>
<td>heat content of products of combustion, kcal/m³</td>
</tr>
<tr>
<td>K_1</td>
<td>A_s/A</td>
</tr>
<tr>
<td>m_a_c</td>
<td>mass of air required for complete combustion of fuel m_F, kg</td>
</tr>
<tr>
<td>m_F</td>
<td>mass of the mixture of combustion gases and air in the fire room, kg</td>
</tr>
<tr>
<td>m_l</td>
<td>initial mass of air in the fire room, kg</td>
</tr>
<tr>
<td>m_a</td>
<td>mass flow rate of air entering fire room, kg/hr</td>
</tr>
<tr>
<td>m_F</td>
<td>rate at which combustible material is being consumed, kg/hr</td>
</tr>
<tr>
<td>m_F_max</td>
<td>maximum rate at which material is being consumed, kg/hr</td>
</tr>
<tr>
<td>m_F_o</td>
<td>initial rate at which material is being consumed, kg/hr</td>
</tr>
<tr>
<td>m_o</td>
<td>mass flow rate of gases leaving fire room, kg/hr</td>
</tr>
</tbody>
</table>
of the model of the room where material is being consumed at a rate \( \dot{m}_F \). Air flows into the room at a rate \( \dot{m}_a \) and combustible gases flow out of the room at a rate \( \dot{m}_o \). The heat transfer to the walls of the room is taken into account as Odeen\(^1\) has shown that it is important in determining the variation of room temperature with time. It is also assumed that the temperature in the room is uniform and that the enthalpy of the combustion products and air at ambient temperature is zero.

The energy balance for the assumed model is:

\[ \dot{H}_{\text{comb}} + \dot{H}_{\text{m}} = \dot{H}_f + Q + \dot{H}_{\text{m o}} \]  \hspace{1cm} (1)

Consider each of the terms in Equation 1.

\( \dot{H}_{\text{comb}} \) is the rate of energy supplied by the combustion of the fuel and is given by

\[ \dot{H}_{\text{comb}} = m_F H \]  \hspace{1cm} (2)

where \( H \) is the effective heating value of the fuel. To take into account the fire spread during the initial phases of the fire, the fuel burning rate was considered to be

\[ m_F = a + b t \quad 0 \leq t \leq t_1 \]  \hspace{1cm} (3)

\[ m_F = m_{F \text{max}} \quad t \geq t_1 \]  \hspace{1cm} (4)

The constants in Equation (3) can be shown to be equal to

\[ a = \dot{m}_{F 0} \quad b = \frac{m_{F \text{max}} - \dot{m}_{F 0}}{t_1} \]  \hspace{1cm} (5)

where \( \dot{m}_{F 0} \) is the burning rate at \( t = 0 \).

\( \dot{H}_{\text{m}} \) is the enthalpy of the air entering the fire room. This term is equal to zero due to the assumption that the enthalpy of air at ambient temperature is equal to zero.

\( \dot{H}_f \) is the rate of change of the enthalpy of the gases in the fire room and is given by

\[ \dot{H}_f = \frac{d}{dt} (H_f) = \frac{d}{dt} (m_f h_f) \]  \hspace{1cm} (6)

where \( m_f \) is the mass of the mixture of combustion gases and air in the fire room and \( h_f \) is the enthalpy per unit mass of the mixture. In the temperature range 0–1500°C (32–2732°F), Odeen recommends the following relationship for the combustion products of solid and liquid fuels:

\[ I = 0.3733 T_f \]  \hspace{1cm} (7)

where \( I \) is the heat content of the products of combustion (k cal/m³) at the temperature \( T_f \) (°C). Therefore, the enthalpy of the combustion gases is given by

\[ H_{cg} = I G_o m_F \]  \hspace{1cm} (8)

where \( G_o \) is the volume of gases produced by the combustion of the fuel (m³/kg). Assuming a mean value of the coefficient of specific heat for the combustion gases, \( C_p \), the enthalpy of the combustion gases can be also written as
\[ \dot{H}_{cg} = (m_F + m_{ac}) C_p T_f \]  
\[ (9) \]

where \( m_{ac} \) is the mass of air required for the complete combustion of the fuel. Equating Equations 8 and 9 and using the definition of \( I \) from Equation 7, the mean value of the coefficient of specific heat at constant pressure if given by

\[ C_p = \frac{0.3733 m_F G_0}{(m_F + m_{ac})} \]  
\[ (10) \]

Table 2 presents the characteristics of the combustion products of wood and fuel oil [1].

<table>
<thead>
<tr>
<th>Fuel</th>
<th>( G_0 )</th>
<th>( \dot{m}_{ac}/\dot{m}_F )</th>
<th>( C_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood ( F = .1 )</td>
<td>5.10</td>
<td>5.45</td>
<td>.295</td>
</tr>
<tr>
<td>( F = .2 )</td>
<td>5.00</td>
<td>5.21</td>
<td>.300</td>
</tr>
<tr>
<td>Fuel Oil</td>
<td>12.00</td>
<td>13.84</td>
<td>.301</td>
</tr>
</tbody>
</table>

Since the value of \( C_p \) for air varies from .24 at 60°F to .28 at 1600°F, it will be assumed that the coefficient of specific heat is independent of temperature. With the assumption of a mean coefficient of specific heat for the mixture of combustion products and air in the fire room, Equation 6 can be written as

\[ \dot{H}_f = \frac{d}{dt}(\dot{m}_r C_p T_f) \]  
\[ (11) \]

\[ = C_p m_r \frac{dT_f}{dt} + C_p T_f \frac{d\dot{m}_f}{dt} \]

The conservation of mass equation for the fire room is

\[ \frac{d\dot{m}_r}{dt} = \dot{m}_a + \dot{m}_F - \dot{m}_o \]  
\[ (12) \]

In addition, since the pressure in the room remains fairly constant, the mass of gas in the fire room is related to the temperature of the room by

\[ m_r = m_i \frac{T_i + 273}{T_f + 273} \]  
\[ (13) \]

where \( m_i \) is the initial mass of air in the room at the temperature \( T_i(°C) \).

\( \dot{Q} \) is the total heat transfer rate through the walls of the room by convection, conduction and radiation, and through the windows by radiation. The method of calculation of \( \dot{Q} \) will be discussed in a later section.

\( \dot{H}_{m_o} \) is the enthalpy of the gases leaving the room and is given by

\[ \dot{H}_{m_o} = \dot{m}_o C_p T_f \]  
\[ (14) \]
Substituting Equations 2, 11, 12 and 14 into Equation 1, one obtains

\[
\dot{m}_f H = \bar{C}_p m_r \frac{dt_f}{dt} + \bar{C}_p T_f (\dot{m}_a + \dot{m}_F - \dot{m}_o) + \dot{Q} + \dot{m}_o \bar{C}_p T_f
\]  

(15)

Solving Equation 15 for the variation of the temperature in the fire room with time, we obtain

\[
\frac{dT_f}{dt} = \frac{1}{\bar{C}_p m_f} \left\{ \dot{m}_f \dot{H} - \dot{Q} - \bar{C}_p [\dot{m}_a + \dot{m}_F] T_f \right\}
\]  

(16)

The heat transfer to the wall is computed by assuming that the wall is divided into a number of slab elements and establishing the heat balance for each slab

![Figure 2. Wall temperature distribution.](image)

(Figure 2). The heat balance equations are derived in Appendix I and have the following form

\[
\frac{dT_1}{dt} = \frac{1}{C \gamma \Delta x} \left[ \alpha - \frac{\lambda}{\Delta x} (T_1 - T_2) \right]
\]

\[
\frac{dT_2}{dt} = \frac{1}{C \gamma \Delta x} \left[ \frac{\lambda}{\Delta x} [T_1 - 2T_2 + T_3] \right]
\]

\[
\frac{dT_n}{dt} = \frac{1}{C \gamma \Delta x} \left[ \frac{\lambda}{\Delta x} [T_{n-1} - T_n] \right]
\]

(17)

The heat balance equation for the \(n\)th slab assumes that there is no heat
transferred out of the slab. (i.e., $T_n = T_{n-1}$) The heat transferred to the wall is therefore given by

$$\dot{Q}_{\text{wall}} = A_s (T_f - T_1) \alpha$$  \hspace{1cm} (18)

The heat transfer coefficient, $\alpha$ is based on conduction, convection, and radiation heat transfer to the surface of the walls. The heat transfer coefficient due to conduction and convection is assumed to be a constant $\beta$ and the radiation heat transfer is based on the Stefan-Boltzmann’s law. The heat transfer coefficient can be expressed as

$$\alpha = \frac{4.96 \varepsilon}{T_f - T_1} \left[ \left( \frac{T_f + 273}{100} \right)^4 - \left( \frac{T_w + 273}{100} \right)^4 \right] + \beta$$  \hspace{1cm} (19)

where $\varepsilon$ is the emissivity between the hot gas mixture and the wall surface. Representative values of the thermal conductivity $\lambda$ (kcal/m hr°C) and the heat storing capacity $C_\gamma$ (kcal/m³°C) are shown in Table 3.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\lambda$</th>
<th>$C_\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>1.3</td>
<td>530</td>
</tr>
<tr>
<td>Brick</td>
<td>.6</td>
<td>450</td>
</tr>
<tr>
<td>Mineral wool</td>
<td>.2</td>
<td>20</td>
</tr>
</tbody>
</table>

The heat transfer radiated through the windows of the room can be determined by the Stefan-Boltzmann law. The heat transfer radiated through the windows is given by

$$\dot{Q}_{\text{wind}} = A_w (T_f - T_1) \alpha_1$$  \hspace{1cm} (20)

where

$$\alpha_1 = \frac{4.96 \varepsilon}{(T_f - T_1)} \left[ \left( \frac{T_f + 273}{100} \right)^4 - (2.73)^4 \right]$$  \hspace{1cm} (21)

The total heat transfer out of the room is given by

$$Q = A(T_f - T_1) [K_1 \alpha + (1 - K_1)\alpha_1]$$  \hspace{1cm} (22)

where

$$A = A_w + A_s$$

$$K_1 = A_s / A$$  \hspace{1cm} (23)
The analysis described in the previous section was programmed for the NYU CDC 6600 computer. The differential equations describing the variation of temperature in the room and in the wall were integrated using a Runge-Kutta technique subject to the following boundary condition

\[ T_f = T_1 = T_2 = \cdots = T_n = 0 \]  \hspace{1cm} (24)

The initial series of computer runs were made to determine the effect of various parameters on the temperature distributions. The following parameters were chosen as representative of conditions for a typical room in which a fire would occur:

- \( A = 481 \, \text{m}^2 \)
- \( C_T = 530 \, \text{kcal/m}^3\text{oC} \)
- \( C_p = .28 \, \text{kcal/kg} \cdot \text{oC} \)
- \( H = 3700 \, \text{kcal/kg} \)
- \( K_i = 1.0 \)
- \( \dot{m}_a = 5.45 \dot{m}_f \, \text{kg/hr} \)
- \( \dot{m}_1 = 603 \, \text{kg} \)
- \( \lambda = 1.3 \, \text{kcal/m hr} \cdot \text{oC} \)

The wall was assumed to consist of five slabs \((n = 5)\) each with a wall thickness of \( \Delta x = .02 \) meters for a total wall thickness of .1 meters. Figure 3 presents the variation of room temperature with time for constant burning rates of 4500 kg/hr and 6000 kg/hr. In Figure 3a the effect of the conduction and convective heat transfer rate, \( \beta \), is shown for the case of zero radiation heat transfer (i.e., \( e = 0 \)). For the care of \( \beta = 0 \) (i.e., \( Q = 0 \)), the equilibrium temperature is independent of the burning rate and equal to 3720°F. Increasing \( \beta \) decreases the room temperature as heat is being transferred out of the room. Figure 3b shows the effect of \( \beta \) for the case of \( e = 0.6 \). As can be seen from the Figure, the effect of conduction and convection heat transfer is small in comparison to the radiation heat transfer because of the high room temperatures.

In Figure 3, the fuel burning rate was assumed constant. In an actual fire the burning rate increases from a low initial value to a maximum burning rate when the entire room is engulfed in flames. In the analyses a linearly increasing burning rate was assumed to represent the fire spread in the room up to the time \( t_1 \). For times greater than \( t_1 \), the burning rate was assumed constant and equal to its maximum value. Figure 4 presents the variation in room temperature as a function of time for several fire spread rates. As can be seen in the Figure, for times greater than \( t_1 \), the difference in room temperature for the fire spread rates considered is less than 100°F.
As stated previously, the heat transfer to the wall is computed assuming that the wall is divided into a number of slab elements and establishing the heat balance for each slab. As a result the temperature is constant in a given slab element. In the actual case, there is a continuous temperature distribution in the
Figure 3b. Variation of room temperature with time for constant fuel burning rates, $c = 0.6$.

Wall material and the wall surface temperature lies somewhere between the temperature in the fire room and the temperature in the first slab as determined by the analysis. Figure 5 shows the effect on the room temperature for two extremes of the wall surface temperature. In the first case the wall surface
Figure 4. Effect of fire spread rate on room temperature. $m_{F_{MAX}} = 6000$ hg/hr.

temperature is assumed equal to the temperature of the first slab, while in the second case the wall surface temperature is assumed to be the average of the temperature in the fire room and the temperature of the first slab. As can be seen in the Figure, the maximum difference in the room temperature is on the
Figure 5. Effect of wall thickness and wall surface temperature on room temperature.

100°F when assuming the two values of the wall surface temperature. Also shown in Figure 5 is the effect of wall thickness on the room temperature.

In the previous calculations, it was assumed that only sufficient air was available for the complete combustion of the fuel (i.e., \( \dot{m}_a/\dot{m}_F = 5.45 \)). Figure 6
Figure 6. Effect of excess air on room temperature.

shows the effect of excess air on the room temperature. For the case shown for the Figure, the room temperature 30 minutes after combustion starts to decrease from 1423°F for 0% excess air to 1264°F for 80° excess air.

The fuel characteristics are specified by the effective heating value of the fuel, H, and the mean coefficient of specific heat at constant pressure of the products
Figure 7. Effect of the effective heating value of the fuel on room temperature.

All of the results previously discussed correspond to \( H = 3700 \) and \( \bar{C}_p = .28 \). Figure 7 shows the effect on the room temperature of the effective heating value of the fuel. The effect of the mean coefficient of specific heat on the room temperature is presented in Figure 8.

The effect of wall material on room temperature is shown in Figure 9. Due to
Figure 8. Effect of mean coefficient of specific heat of combustion products on room temperature.

The low thermal conductivity and heat storage capacity of mineral wool, the temperature of a room with walls made of mineral wool is almost double of what it would be if the walls were made of concrete.

Figure 10 presents the variation of room temperature as a function of room window area. As can be seen in the Figure, the room temperature is less than
Figure 9. Effect of wall material on the room temperature.

100°F different when the room contains 20% of the surface area as windows (i.e., $K_1 = .8$) in comparison to all concrete surfaces.

Conclusions

The results presented in the previous section clearly indicate the importance of various parameters in the temperature build up in a fire room. It is important
Figure 10. Effect of window area on room temperature.

that these conditions be known for a fire test if an attempt is to be made to compare the experimental results with theoretical results.

REFERENCES

APPENDIX I. HEAT BALANCE EQUATION

Figure 2 presents a schematic of the wall slab elements used in the heat transfer analysis. The heat balance equations for the slab elements are given by

\[
\frac{d T_1}{d t} = (C \gamma \Delta x) \left( \frac{dT_1}{dT} \right) = q_w - q_{12}
\]

\[
\frac{d T_2}{d t} = (C \gamma \Delta x) \left( \frac{dT_2}{dT} \right) = q_{12} - q_{23}
\]

\[
\frac{d T_3}{d t} = (C \gamma \Delta x) \left( \frac{dT_3}{dT} \right) = q_{23} - q_{34}
\]

\[
\frac{d T_n}{d t} = (C \gamma \Delta x) \left( \frac{dT_n}{dT} \right) = q_{n-1,n} - q_{n,n+1}
\]

where \(q_w\) is the heat transfer rate kcal/m² sec from the fire room to the first slab element and \(q_{n-1,n}\) is the heat transfer rate from the \((n-1)\)th slab to the \(n\)th slab.

The heat transfer from the hot gases in the fire room to the first slab element is due to radiation and convection.

\[q_w = \alpha (T_f - T_i)\]

where

\[
\alpha = \frac{4.96 \epsilon}{T_f - T_i} \left[ \left( \frac{T_f + 273}{100} \right)^4 - \left( \frac{T_w + 273}{100} \right)^4 \right] + \beta
\]

The heat transfer between slab elements is due to conduction and is given by

\[q_{12} = \frac{\lambda}{\Delta x} [T_1 - T_2]\]

\[q_{n-1,n} = \frac{\lambda}{\Delta x} [T_{n-1} - T_n]\]

Assuming that the outside wall of the \(n\)th element is insulated, that is \(q_{n,n+1} = 0\), and substituting the definitions of the heat transfer rates into the heat balance equations, one obtains

\[
\frac{dT_1}{dt} = \frac{1}{C \gamma \Delta x} \alpha - \frac{\lambda}{\Delta x} (T_1 - T_2)
\]

\[
\frac{dT_2}{dt} = \frac{1}{C \gamma \Delta x} \frac{\lambda}{\Delta x} [T_1 - 2T_2 + T_3]
\]

\[
\frac{dT_n}{dt} = \frac{1}{C \gamma \Delta x} \frac{\lambda}{\Delta x} [T_{n-1} - T_n]
\]