EVALUATION OF COSTS ASSOCIATED WITH REGIONAL, ENVIRONMENTAL IMPACT IN CHESAPEAKE BAY

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ABSTRACT
In this article, environmental impact in Chesapeake Bay is examined, particularly as it relates to the health and maintenance of Striped Bass populations valuable to both sports and commercial fisheries. A dynamic simulation model is developed using optimal control theory to calculate accumulating opportunity costs of fish population decline caused by pollution in the Bay. The model is then applied to the actual situation using data on Striped Bass populations and rates of habitat loss resulting from disappearing submerged aquatic vegetation. The model is set up to provide information that can be used in benefit-cost analysis, and the impact cost is calculated over time periods during which environmental conditions are modeled to change.

INTRODUCTION
Declining environmental quality affects both producers and consumers, and forces environmental planners to make decisions about acceptable levels of pollution. This is not a trivial task, for the relationships between the environment, production, and consumption can be exceedingly complex. One ingredient to making an informed decision is an assessment of the costs associated with environmental degradation. Once such an assessment is made, it is a straightforward matter to compare costs and benefits. If the environmental
costs, translated into dollars, are greater than the benefits associated with a proposed development or activity, a planner can justify environmental preservation on the grounds of economic efficiency. Of course, should benefits outweigh costs, values other than environmental preservation are more important.

Kahn notes that there have been few published studies in which ecological and economic theory have been combined into planning models [1]. This is perhaps surprising given that an ecosystem is often the crucial link between a polluting activity and a perceived impact. Declining water quality, for example, may be how recreational users of a lake view the change, brought about by agricultural runoff, from a clean water ecosystem to one that is subject to blue-green algal blooms. Associated with various levels of agricultural runoff are specific groupings of flora and fauna, some more agreeable to the human senses than others. As the nutrient levels in the runoff increase, the ecology generally changes for the worse, at least from a user's point of view. For many types of pollution impact a direct relationship of this kind exists between the polluting activity and the attributes of an ecosystem which underpin environmental quality. This article presents one way to incorporate information about such relationships into a benefit-cost framework.

Chesapeake Bay has recently been the subject of intensive, scientific investigation, inspired partly by a widespread impression that its environmental quality has been deteriorating [2-4]. Since the 1960s, significant ecological changes have occurred there, and the public, for the most part, has found them unappealing. One of the major changes is the loss of underwater plants, or submerged aquatic vegetation (SAV). Typically, such plants have been associated with relatively clean water and sizeable populations of quality fish, for which they serve as a nursery habitat. Free-floating phytoplankton have to a large extent replaced the SAV as the predominant, photosynthetic component of the estuarine ecosystem. Evidently, this effect has been brought about by declining water quality in the Bay caused by nonpoint, nutrient-rich runoff.

Although the decline in the Bay's water quality is widely recognized, the related costs are unknown. Many fisheries, both shellfish and finfish, are supported by the Bay ecosystem, and it is often difficult to ascertain the effect of ecological disruption on particular species. Moreover, commercial and sports fisherman may value the same species in different ways. Aside from these complexities are those associated with option values which arise from the existence of the Bay as a natural environment, and recreation values deriving from the availability of clean water inviting to swim in. Further, environmental impact in the Chesapeake Bay is not a once-and-for-all change in the Bay's ecology, leading to a unique and enduring loss of value. Rather, this impact is the result of a long-term trend of nutrient enrichment.

In this article, a model is presented which links an ecological process with an economic activity. It is then applied by way of illustration to the Chesapeake
Bay. The model is specifically designed to reflect the dynamic nature of many types of pollution impact, particularly those resulting from regional growth and change. A coastal area such as that which surrounds Chesapeake Bay is embedded in a large, physiographic region including parts of Maryland, Virginia, West Virginia, Pennsylvania, Delaware, and New York. Pollution in the Bay cannot easily be attributed to a single cause or location. Nonpoint runoff seeps from the coastline, draining many square miles of agricultural and residential land. Moreover, there are numerous coastal communities with individual point sources of pollution, and it is very difficult, if not impossible, to know with certainty what any one of them is contributing to the overall problem in the Bay. As land use changes occur in the Bay’s watershed and as the population there grows, estuarine water quality is affected over the long-run.

Benefit-cost analysis is frequently undertaken in an equilibrium framework, and the costs and benefits calculated are presented as annual values which can be discounted and summed if the analyst so desires. Such an approach is adequate if an equilibrium prevails, and values remain constant over time. Yet should an environmental effect grow more severe, or should natural amenities appreciate, to simply discount and sum the same value provides an underestimate of environmental costs [5, pp. 149-153]. The model presented here, provides a way for assessing costs which stem from a long-run impact of variable intensity.

THE MODEL

The Schaefer model of the single-species fishery poses a relationship between equilibrium outputs and the effort expended by fisherman to obtain them [6]. The model is founded on the principle of logistic growth first stated by Verhulst in 1838 [7]. The dynamics of logistic growth are determined by the intrinsic rate of growth of the species and by the carrying capacity of the environment. The stock of a population will approach the carrying capacity value if given enough time to do so. The Schaefer model is derived by including a harvest function in the differential equation of logistic growth and solving for equilibrium yields.

The ecological process modeled in this article is the reduction in the carrying capacity of a fish species due to a continuing decline in environmental quality [8]. It is assumed here that the species in question is commercially exploited, so harvest is affected by changes in its carrying capacity. As pollution levels increase with time, the environment’s carrying capacity for the species is continuously eroded. This then tends to reduce the harvest during any given time period. As the value of total production decreases, society bears an opportunity cost in the form of lost surpluses to both producers and consumers.

In this article, the commercial fishery for Striped Bass in Chesapeake Bay is modeled. The Striped Bass population is one of the most important renewable resources of the Bay, and until the fishery was closed in 1985 due to seriously
declining populations, harvest of this species provided a major source of value to area residents. The model presented here is based on the theory of optimal exploitation of renewable resources and is therefore most applicable to commercial fisheries in the Bay, but not to the sports fisheries, which, in the case of Striped Bass, was considerable prior to 1985.

Optimal harvest policy for a renewable resource is well-known [7, 9-12], but has not been closely examined for the case in which the resource is under environmental stress. In that declining environmental quality will adversely affect an exploited population and reduce its standing stock, the optimal harvest schedule will be different than when no environmental effect occurs. This study considers some of the economic consequences of environmental impacts on renewable resources by reformulating the traditional, dynamic, optimization problem addressed in the literature. In the traditional problem, an owner of a renewable resource chooses a harvest rate which maximizes the present value of the natural commodity over an infinite time horizon. For the duration of harvest, the parameters of the ecological system do not change. In the solution given here, the productivity of the ecological system is influenced by conditions in the environment, which are allowed to change.

The agent of choice in the traditional problem is an owner who exercises property rights over the resource and is therefore able to exclude other potential resource users. This institutional setting is at odds with certain aspects of the economic theory of the fishery, in that with common property resources no one user possesses the legal power to exclude any other user. This situation, one of open access, leads to the dissipation of rents, for so long as positive profits prevail additional users are drawn to the resource industry [13]. If there are no barriers to entry, new users will continue to be attracted until profits vanish.

In fact, fishery resources are not always a pure common property in the strict sense of the term. National governments frequently exercise a broad jurisdiction over coastal waters and limit the entry of foreign vessels. Close to shore, informal property relations may spring up as they have among some harvesters in the lobster industry [14]. More formally, oyster beds may be leased in Virginia, and gillnet anchoring sites are sold for salmon fishing in Bristol Bay, Alaska [15]. In many instances, a government office acts as a property owner and regulates the entry of local fishermen. This is frequently done by selling licenses. In Chesapeake Bay, for example, the Maryland Department of Natural Resources charges $250.00 per year for the privilege of using a large gill net in state waters [16]. Moreover, the State possesses sufficient legal power to entirely ban the harvest of threatened species, as it has done for Striped Bass.

An institutional environment of this kind is a significant departure from that of unregulated, open access. In such a setting, the regulatory agency exercises a form of property right and even appropriates resource rents. Under purely open access all rents are dissipated and there is no producer surplus. With regulation,
a surplus is obtained by the regulatory agency, and this surplus may be reallocated to society. The economic agent of the model presented here is perhaps best interpreted as a State agency empowered to sell licenses and control harvest rates.

Price of the natural commodity enters as an exogenous variable in the traditional optimization problem. For renewable resource goods this may or may not be a realistic assumption [17]. Some fishery products, like the Maine lobster, are relatively unique, and the production in one locality serves a very large market. In such a situation, the demand curve confronted by the industry is not elastic, and demand prices may vary substantially with the level of harvest. For other fishery goods, demand is highly elastic in individual localities, where variable production does little to influence market price.

A demand analysis of Striped Bass in Chesapeake Bay shows that at a significance level of .01 an assumption of perfect price elasticity cannot be rejected [1]. Thus, the price assumption in the traditional model is adopted for the application presented here (for some of the other fisheries in the Bay, notably that for oysters, it would probably be necessary to incorporate a price effect). Under the constant price assumption, consumer surplus does not exist, at least over the domain of potential harvests from the regional fishery. Hence, environmental impact does not create a loss to consumers via increasing prices, although incomes may be reduced if the resource rents reallocated to society are large enough. In the sequel, other price structures will be considered, and it will be suggested how consumer surplus might be incorporated into the model.

In the discussion which follows, environmental impact is first incorporated into the traditional, optimal control problem. Next, a simulation methodology is developed for the case in which overexploitation of a renewable resource occurs in the initial period. A simulation is then undertaken for Striped Bass in Chesapeake Bay, and results are presented for two cases: with and without environmental impact. Finally, it is shown how these two sets of results may be used to give a measure of costs attributable to the impact.

THE OPTIMIZATION PROBLEM

The problem to be solved is that of an individual or institution which wishes to maximize profits from harvesting a renewable resource good. Although in the traditional problem harvest is assumed costless and the natural commodity is taken as numeraire, here cost is assumed to be linear in harvest and a constant price is associated with the good. The difference between price and cost is then the profit per unit of output, or net revenue [18].

The stock of the renewable resource is described by a growth function inducing logistic growth dynamics. Harvest rates belong to a constrained set with an upper and lower bound. In the application, the lower bound is taken
as zero, for no harvest need occur at all. The maximization problem may be stated as:

$$\begin{align*}
\text{maximize} & \quad \int_0^\infty (p-c) y(t)e^{-rt} dt \\
\text{subject to} & \quad \dot{x} = F(x) - y(t), \\
& \quad y(t) \in [0, y_{\text{max}}], \\
& \quad x_0 = x(0), \text{ where} \\
& \quad r = \text{discount rate}, \\
& \quad x = \text{resource stock}, \\
& \quad \dot{x} = \frac{dx}{dt}, \\
& \quad y = \text{harvest rate}, \\
& \quad p = \text{price}, \\
& \quad c = \text{unit cost}, \\
& \quad y_{\text{max}} = \text{maximum harvest rate}, \\
& \quad x_0 = \text{initial stock}, \text{ and} \\
& \quad F(x) = \text{growth function}.
\end{align*}$$

As is well-known, the solution to this problem consists of most-rapid approach paths [9]. If the stock is initially underexploited, optimality requires maximum harvest until the singular path is reached. Conversely, given initial overexploitation, it is optimal to forego harvest until the singular path is obtained. The singular path in this problem is a unique stock level given implicitly by the equation,

$$F'(x) = r. \quad (1)$$

It is assumed in the development which follows that overexploitation prevails in the first time period. In this way, a worst case scenario is anticipated whereby environmental impact stresses a resource already heavily exploited. An estuarine fishery—such as for Striped Bass in Chesapeake Bay—located near a major population area is an example of an industry dependent on resources so affected. Solutions to the overexploited case entail an initial moratorium during which it is optimal to conserve resources.

Note that in this formulation of the problem, cost is taken as linear in harvest, and there is no effect whereby costs increase as the stock of the species declines. If pollution in the Bay leads to a loss of habitat, the range of Striped Bass diminishes—as does the carrying capacity and the stock—but the density of the species need not change in unaffected areas. So long as fishermen have adequate information about what parts of the Bay are unpolluted, unit cost of harvesting need not be seriously affected by declining environmental quality.
Environmental impact affects the problem by altering the specification of the growth function of the harvested species, which is parameterized in environmental carrying capacity and the population's intrinsic rate of growth. If it is assumed that environmental impact reduces carrying capacity over time, then carrying capacity is variable in time; this makes the growth function variable in both time and stock. A modified growth function may then be given as:

\[ F(x,t), \]

and the original problem becomes:

\[
\text{maximize} \quad y \\
\text{subject to} \quad \dot{x} = F(x,t) - y(t), \\
\quad y(t) \epsilon [0, y_{\max}], \\
\quad x_0 = x(0).
\]

It can be shown that the solution to this problem is similar to the first one stated in which the growth function is independent of time. However, the singular path is now a function in time and no longer a unique stock level as in the original case. The singular path under environmental impact is given by the function (see [7]):

\[ F_x(x,t) = r, \text{ where} \]

\[ F_x = \frac{\partial F}{\partial x}. \]

If environmental impact grows increasingly severe, stocks may be entirely eliminated, in which case harvest no longer occurs. Hence,

\[
\int_0^\infty (p-c) y(t)e^{-rt}dt = \int_0^\beta (p-c) y(t)e^{-rt}dt, \text{ where} \\
\beta = \text{time stock vanishes.}
\]

The determination of \( \beta \) is undertaken in the sequel.

**PRESENT VALUE UNDER ENVIRONMENTAL IMPACT**

Harvest may be given as

\[ y(t) = F(x,t) - \dot{x}. \]

Since the singular path may be directly obtained via (2) as

\[ x(t) = x^*(t), \]
harvest along the singular path is
\[ y(t) = F(x^*(t), t) - \dot{x}*. \]

It is known, \textit{a priori}, that the optimizing \( x(t) \) is given by
\[ F_x(x, t) = r. \]

In the methodology this is not assumed, but ascertained through simulation. To this end, many paths may be identified by
\[ F_x(x, t) = \psi, \quad \text{where} \quad \psi = \text{some arbitrary constant}. \]  

It is known, therefore, that optimization occurs when (3) is solved for \( \psi = r \).

A most rapid approach path consistent with a solution to (3) is composed of two harvest schedules. In that overexploitation is assumed as an initial condition, harvest does not occur at first, or
\[ y(t) = 0 \quad 0 < t < \Theta. \]

At time \( t = \Theta \), harvest begins. This is the time at which (3) solved, or
\[ F_x(x, \Theta) = \psi, \quad \text{and} \]
\[ F_x(x, t) = \psi \quad \text{for} \quad 0 < t. \]

Hence, the second harvest schedule is that which sustains the solution to (3). The overall harvest policy is therefore
\[ y(t) = 0 \quad \text{for} \quad 0 \leq t < \Theta, \]
\[ y(t) = y^{**}(t) \quad \text{for} \quad \Theta \leq t, \quad \text{where} \]
\[ y^{**}(t) = \text{harvest implied by (3)}. \]

The present value (PV) of the harvest for an arbitrary most rapid approach path is:
\[
PV = \int_{0}^{\infty} (p-c) y(t)e^{-rt}dt, \\
= \int_{\Theta}^{\infty} (p-c) y^{**}(t)e^{-rt}dt.
\]

The simulation methodology consists of examining the present value of all possible most rapid approach harvest policies. This is done by simulating the growth of the population stock. In the absence of harvest, stock accumulates in direct response to the growth function. At each instant in time as growth occurs, there exists a unique partial derivative of the growth function with respect to stock. Therefore, associated with some time \( \Theta \) is a value of the partial derivative, namely \( \psi \). A stock path which sustains the value of the partial derivative may be calculated via (3). The stock path so computed may be used
to determine harvest rates which sustain the constant value of the derivative. In turn, the present value of these various harvests may be ascertained. Since no harvest occurs until \( \Theta \), and since harvest sustains condition (3) beyond \( \Theta \), the present value is that of a most rapid approach path, from below, to a harvest which fulfills the condition \( F_x(x,t) = \psi \). As stated, the optimizing most rapid approach path occurs when \( \psi = r \). The simulation methodology consists of searching along an unharvested growth trajectory for the optimal present value.

At each instant in time as unharvested stock accumulates, the partial derivative is calculated and the associated stock path derived. This enables the calculation of the present value as a continuous function of time. Since a present value is associated with all possible stock levels throughout the entire growth trajectory, the search is exhaustive, and the optimal harvest is brought to light.

The logistic growth function is frequently specified as:

\[
F(x) = ax - bx^2, \quad \text{where}
\]

- \( x = \text{stock} \),
- \( a = \text{intrinsic rate of growth} \),
- \( b = \frac{a}{k} \), and
- \( k = \text{carrying capacity} \).

In the treatment presented here, carrying capacity is modeled as a function of environmental quality, which is assumed to deteriorate through time in a linear fashion. Thus, carrying capacity is a function of time, or

\[
k(t) = d - ft, \quad \text{where}
\]

- \( d = \text{initial carrying capacity} \),
- \( f = \text{rate of environmental degradation} \), and
- \( t = \text{time} \).

The growth function used in the methodology is therefore,

\[
F(x,t) = ax - b(t)x^2, \quad \text{where} \quad b(t) = \frac{a}{k(t)}. \tag{4}
\]

The parameters \( d \) and \( f \) may be fit with field data. It should be noted that the linear specification is arbitrary; other functions may be easily incorporated into the approach.

The partial derivative in \( x \) of (4) is

\[
F_x(x,t) = a - 2b(t)x. \tag{5}
\]
If this partial derivative is set at some arbitrary value $\psi$, a function of $x$ in time is implied, or

$$x(t) = \frac{a-\psi}{2b(t)}.$$  \hfill (6)

This may be differentiated,

$$\dot{x}(t) = \frac{(\psi-a)}{2b(t)^2} \dot{b}(t).$$  \hfill (7)

The available harvest when stock is observed to follow the function in (6) is

$$y(t) = F\left[\frac{a-\psi}{2b(t)} + \frac{(a-\psi)\dot{b}(t)}{2b(t)^2}\right].$$  \hfill (8)

The present value of the harvest schedule given by (8) is

$$PV = \int_\Theta^\infty (p-c) \left\{ F\left[\frac{a-\psi}{2b(t)} + \frac{(a-\psi)\dot{b}(t)}{2b(t)^2}\right] e^{-rt} dt \right\}. \hfill (9)$$

As stated, the time $\Theta$ is when the marginal growth rate reaches the value $\psi$ at which it remains.

When carrying capacity is modeled as a linearly decreasing function, stock vanishes at time $= d/f$, since

$$x(t) = \frac{a-\psi}{2b(t)} \text{, or}$$
$$x(t) = \frac{a-\psi}{2} \frac{d-f}{a} \text{, and}$$
$$x(t) = 0 \Rightarrow t = d/f.$$  

Hence, the present value integral (9) may be solved as (see Appendix i):

$$PV = Q \left[ \frac{e^{-r\Theta}}{r} - e^{-r(d/f)} \right] + Z \left[ \frac{e^{-r\Theta}}{r} (1+r\Theta) - e^{-r(d/f)} (1+r(d/f)) \right],$$

where

$$Q = (p-c) \left[ \frac{(a-\psi)^2}{4a} f - \frac{(a-\psi)^2}{2} f \right],$$

$$Z = (p-c) \left[ \frac{(a-\psi)^2}{4a} f - \frac{(a-\psi)^2}{2} f \right],$$

$\Theta =$ time that harvesting begins.

Since overexploitation is assumed in the initial time period, stock accumulates from the beginning as,

$$\dot{x} = F(x,t).$$  \hfill (10)
At any instant in time harvest may start; associated with this time \( \Theta \) is a partial derivative,

\[
F_x (x, \Theta) = \psi.
\]

The simulation methodology is an algorithm which generates logistic growth according to (10). At each instant along the growth trajectory, the present value (9) is calculated for the harvest implied by (3). In this way a function of present values is determined, showing potential profits for the entire trajectory of the population stock. The maximum present value is identified by visual display of the output.

**SIMULATION**

Simulations begin in 1974, a year for which a reliable estimate for Striped Bass carrying capacity exists. A twenty-year environmental effect is assumed. That is, given the specification of the carrying capacity equation, stocks will vanish after twenty years along the singular path, given a particular initial stock level. The growth function used in the simulation, taken from experimental field data,\(^2\) is

\[
x = \frac{x^2}{60,000,000 - 3,000,000t}
\]

where stock, \( x \), is measured in pounds. The initial carrying capacity, \( k(0) \), is thus \( 60 \times 10^6 \) pounds. The initial stock level is arbitrarily set at 50,000 pounds to indicate severe exploitation. A discount rate of .03 is used, as is common practice \([20]\).

In Figure 1, present values are given for the various harvesting policies. If harvesting begins immediately, stocks are low and profits are small. If harvesting is deferred to later dates, greater profits can be obtained. The present value function is spiked, the peak of which identifies both a time and a present value. This time is the optimal duration of the period of no harvest, or the optimal moratorium on exploitation. Once this time is reached, it is optimal to begin harvesting so that stock declines along the locus for which a constant marginal growth rate is maintained. The value of this partial derivative will be the discount rate. In an application, the actual harvest schedule may be calculated directly from (8).

In Figure 2, the locus of Figure 1 is reproduced, and a new function is added. This new function is the present value of a most rapid approach harvest routine under the condition of no environmental impact. The peak of this second spike

\(^2\) This function comes from research conducted into environmental impact in Chesapeake Bay under the auspices of the Chesapeake Bay Program \([19]\). The carrying capacity is a field estimate for 1974. The degradation rate is assumed to follow the loss of submerged aquatic vegetation; as such it is not as severe as that indicated by research \([2, 3]\).
occurs slightly later than the first one for degradation of the environment encourages an accelerated depletion of the natural resource. Since stocks are vanishing, it is optimal to exploit them quickly.

The present value of harvesting under environmental impact is less than when no impact occurs, as diminishing carrying capacity reduces the biotic potential of the population and, hence, potential yields. Comparing the optimal
Figure 2. Present value for the cases with and without environmental impact.

Present values for the two functions in Figure 2 provides a measure of the profit losses induced by environmental impact. If Striped Bass is valued at $.57/pound, this loss, or the opportunity cost of pollution, is approximately $12,570,000.

The exvessel value of 2,648,000 pounds of Striped Bass in 1977, the most recent year of data tabulation, is $1,520,000 which implies a price of $.57/pound [21]. Returns to capital and captain's labor vary substantially across fisheries. For example, in the Gulf of Mexico and South Atlantic Groundfish fishery, 7.4 percent of revenues are paid to capital and the captain. For a Louisiana shrimp boat with a crew of four, the comparable percentage is 36.9 [22]. In that these percentages include the payment to the captain's labor, the return to capital alone is probably much less, which implies that total production costs are a substantial fraction of revenues. We assume conservatively and for the sake of simplicity that profits in the Striped Bass fishery are 5 percent of revenue. Given a price of $.57/pound, profit is approximately $.03/pound.
DISCUSSION

The simulation methodology presented in this article provides an estimate of profit loss in a renewable resource industry brought about by pollution impact in a large estuarine watershed. Although the loss calculated is sizeable in and of itself, it is only one component in a set of similar losses. Other fisheries in Chesapeake Bay are affected by pollution too, as are a number of sports fisheries. Moreover, the costs of pollution in the Bay are by no means restricted to productive industries, for the Chesapeake is host to many different kinds of recreational activities and serves a large population. A complete assessment of the costs associated with declining environmental quality in the Chesapeake would consider the Bay’s many different uses and values. The methodology presented here could provide some of the input necessary to making an overall assessment.

It was assumed in developing the numerical method utilized in the calculations that both price and cost were constant. However, in fisheries other than that for the Bay’s Striped Bass, demand conditions may be such that price varies with harvest rate, thereby invalidating the constant price assumption. In such a situation, consumer surplus should be considered in the impact calculation, for as price increases with diminishing harvest rates, consumers lose a source of value.

To add greater realism to the production process, it would also be useful to include a more complicated cost function. In particular, scale economies could be introduced, as could a stock effect whereby unit costs increase when stock declines. Optimal management of a renewable resource with inelastic demand and nonlinear costs would then involve maximizing the function [7, 23]:

$$\text{maximum}_{y} \int_{0}^{\infty} [U(y) - C(x,y)y] e^{-rt} dt, \text{ where}$$

$$U(y) = \int_{0}^{y} P(y)dy.$$  

In this formulation, $U(y)$ is a social benefit function the value of which is given by integrating a demand curve over rates of output. Cost is represented by a function variable in both resource stock and harvest. As such, this would enable stock effects and scale economies to influence the optimizing pattern of exploitation.

The effect of pollution impact could be modeled in the same way here as in the constant price/constant cost case. That is, a non-autonomous growth function with time-varying carrying capacity could be used to describe the dynamics of the stock variable. Once the optimal exploitation pattern had been established, the indicated harvest rate could be placed in the optimization integral, the value of which could be calculated by numerical methods. Comparing this value to one determined in a similar manner for the case without environmental impact would provide a measure of the losses inflicted by pollution.
\textbf{APPENDIX 1}

\begin{equation}
PV = \int_{\Theta}^{d/\ell} (p-c) \left\{ \frac{a-\psi}{2b(t)} + \frac{(a-\psi)b(t)}{2b(t)^2} \right\} e^{-\theta \cdot dt} = (p-c) \int_{\Theta}^{d/\ell} F \left[ \frac{a-\psi}{2b(t)} \right] e^{-\theta \cdot dt} + \int_{\Theta}^{d/\ell} \frac{(a-\psi)b(t)}{2b(t)^2} e^{-\theta \cdot dt}, \tag{1}
\end{equation}

\begin{align*}
F \left[ \frac{a-\psi}{2b(t)} \right] &= \frac{a}{2b(t)} - \frac{b(t)(a-\psi)^2}{4b(t)^2} \\
&= \frac{a(a-\psi)}{2b(t)} - \frac{(a-\psi)^2}{4b(t)}, \text{ and}
\end{align*}

\begin{align*}
b(t) &= \frac{a}{d-ft}, \\
F \left[ \frac{a-\psi}{2b(t)} \right] &= \frac{(a-\psi)(d-ft)}{2} - \frac{(a-\psi)^2(d-ft)}{2}. \tag{2}
\end{align*}

\begin{align*}
b(t) &= \frac{af}{(d-ft)^2}, \text{ so}
\frac{(a-\psi)b(t)}{2b(t)^2} &= \frac{(a-\psi)(af)}{2(d-ft)^2} \cdot \frac{(d-ft)^2}{a^2} = \frac{(a-\psi)f}{2a}. \tag{3}
\end{align*}

Substituting (2) and (3) into (1) yields

\begin{align*}
PV &= \int_{\Theta}^{d/\ell} (p-c) \left[ \frac{(a-\psi)(d-ft)}{2} - \frac{(a-\psi)^2(d-ft)}{4a} \right] e^{-r \cdot dt} + \\
&\quad \int_{\Theta}^{d/\ell} (p-c) \frac{(a-\psi)f e^{-\theta \cdot dt}}{2a} \\
&= (p-c) \left[ \frac{(a-\psi)d}{2} - \frac{(a-\psi)^2 d}{4a} + \frac{(a-\psi)f}{2a} \right] \int_{\Theta}^{d/\ell} e^{-r \cdot dt} + \\
&\quad (p-c) \left[ \frac{(a-\psi)f^2}{4a} + \frac{(a-\psi)f}{2a} \right] \int_{\Theta}^{\infty} e^{-r \cdot dt}, \text{ and}
\end{align*}

\begin{align*}
PV &= Q \left[ \frac{e^{-r\theta}}{r} - \frac{e^{-r(d/\ell)}}{r} \right] + Z \left[ \frac{e^{-r\theta}}{r^2} \left( 1 + \theta e^{-r(d/\ell)} \right) - \frac{(1+r\theta)e^{-r(d/\ell)}}{r^2} \right],
\end{align*}

where

\begin{align*}
Q &= (p-c) \left[ \frac{(a-\psi)d}{2} - \frac{(a-\psi)^2 d}{4a} + \frac{(a-\psi)f}{2a} \right], \text{ and}
\end{align*}
\[ Z = (p-c) \left[ \frac{(a-\psi)^2 f}{4a} - \frac{(a-\psi)f}{2} \right] . \]

(ii) \[ PV = \int_{\Theta}^{\infty} y(t) e^{-rt} dt \]
= \[ y \int_{\Theta}^{\infty} e^{-rt} dt \]
= \[ y \frac{e^{-r\Theta}}{r} \], where

\( y \) = equilibrium harvest implies by \( \dot{x} = 0 \), or
\( y = F(x) \).

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